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NUMERICAL ANALYSIS OF THE MICRO-POLAR FERROMAGNETIC ROTATING FLUID FLOW SATURATING A POROUS MEDIUM

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Abstract: The numerical analysis of the micro-polar ferromagnetic rotating fluid layer heated from below in the presence of uniform vertical magnetic field in a porous medium has been carried out. The dispersion relation has been analyzed using normal mode and it is found that the permeability has destabilizing effect. The rotation, coupling parameter, heat conduction parameter and micro-polar coefficient have stabilizing effects. The sufficient condition for the non-existence of over stability has been found.

Keywords: Micro-polar fluid; Porous medium; Rotation

1. Introduction

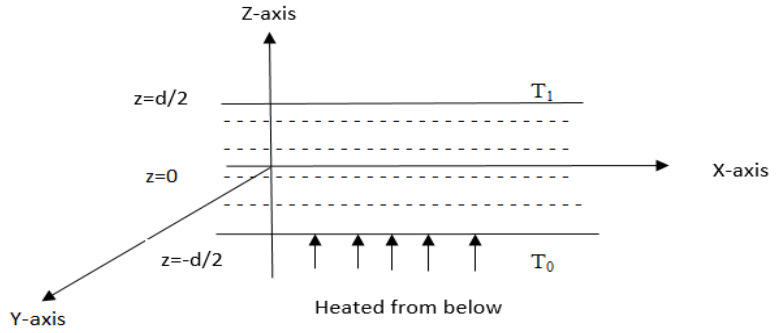
There are some important classes of fluid in the field of technology, one of them being micro-polar ferromagnetic fluid. In order to study micro-polar ferromagnetic fluids, the general theory of micro polar fluid was introduced and micro rotational inertia & micro rotational effects of micro-polar fluid were shown according to Eringen [1, 2]. Qin and Kaloni [9] studied the problem of thermal instability in a rotating micro-polar fluid. Kumar and Mehta [5] demonstrated the effects of permeability, hall parameter, magnetic fields in a porous medium and the sufficient condition for no more stability is as $\beta_e < \frac{1}{6\pi^2 P_m}$.

Wooding [12] discussed the Rayleigh instability of flow through a porous medium. Gupta and Gupta [4] discussed the effect of rotation on the ferromagnetic fluid. Finalayson [3] analyzed the convective instability of ferromagnetic liquids by the Galerkin method. Singh [10] discussed the effect of rotation and suspended particles on micro-polar ferromagnetic fluid. Sunil et al. [11] analyzed the effect different parameters on the micro-polar ferromagnetic fluid. Mittal and Rana [6] investigated numerically and graphically the effect of suspended particles on the micro-polar ferromagnetic fluid. Pant and Algehyne [7] discussed the ferromagnetic fluid layer with changeable gravity and viscosity. Pundir et al. [8] investigated the Maxwell ferromagnetic fluid with thermal convection.

Application of this work is in geophysics, engineering science, chemical science and industry like liquid crystal, blood flows, colloids suspensions and clean engine lubricants. In this paper, we attempt to numerical study of the micro-polar ferromagnetic rotating fluid flow saturating a porous medium. To our knowledge this problem has not yet been investigated using the generalized Darcy's model.

2. Mathematical Formulation

An infinite, horizontal, incompressible electrically non-conducting micro-polar ferromagnetic fluid layer of thickness d is assumed which has porosity $\in$ and medium permeability k_1. The higher boundary $z = d/2$ and lower boundary $z = -d/2$ are maintained at constant but varying temperatures T_0 and T_1 such that a study adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ has been continued. The rotation, gravity and uniform magnetic field is applied along the z-axis to the system.



The equation of continuity, motion, angular momentum, energy and basic state are given as follows:

$$\nabla.\vec{q} = 0 \tag{1}$$

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right] = -\nabla P - \rho g \hat{e}_z + \nabla\left(\vec{H}.\vec{B}\right) - \frac{1}{k_1}(\mu + \varsigma)\vec{q} + 2\varsigma(\nabla\times\vec{v}) + \frac{2\rho_0}{\in}(\vec{q}\times\Omega) \tag{2}$$

$$\rho_0 J\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\in}(\vec{v}.\nabla)\vec{v}\right] = \varsigma\left[\frac{1}{\in}\nabla\times\vec{q} - 2\vec{v}\right] + (\alpha' + \beta')\nabla(\nabla.\vec{v}) + \gamma'\nabla^2\vec{v} + \mu_e\left(\vec{M}\times\vec{H}\right) \tag{3}$$

$$\begin{aligned}\in\left[\rho_0 C_{v,H} - \mu_e\vec{H}\left(\frac{\partial \vec{M}}{\partial T}\right)_{v,H}\right]\left[\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T\right] + (1-\in)\rho_s C_s\frac{\partial T}{\partial t} \\ + \mu_e T\left(\frac{\partial \vec{M}}{\partial T}\right)_{v,H}\left[\frac{\partial \vec{H}}{\partial t} + (\vec{q}.\nabla)\vec{H}\right] = \chi\nabla^2 T + \delta(\nabla\times\vec{v}).\nabla T\end{aligned} \tag{4}$$

$$\rho = \rho_0 \left[1 + \alpha \left(T_a - T\right)\right] \tag{5}$$

where ρ – Fluid density, ρ_0 – Reference density, $\vec{q}$ – Filter velocity, $\vec{v}$ – Spin (micro rotation), μ - Shear kinematic viscosity coefficient, ς – Coupling viscosity coefficient, P– Pressure, μ_e – Magnetic permeability, $\hat{e}_z$ – Unit vector in z-direction, α'– – Bulk spin viscosity coefficient, β' – Shear spin viscosity coefficient, γ' – Micro-polar viscosity coefficient, J–Micro inertia constant, t–time, $C_{v,H}$ – Specific heat at constant volume and magnetic field, C_s – Specific heat of solid (Porous Material Matrix), ρ_s – Density of solid matrix, χ – Thermal conductivity, T–Temperature, δ – Micro-polar heat conduction coefficient, α – Coefficient of thermal expansion, $\vec{B}$ – Magnetic induction, $\vec{M}$ – Magnetization.

The Maxwell's equations are taken as follows:

$$\nabla.\vec{B} = 0 \, and \, \nabla \times \vec{H} = 0 \tag{6}$$

$$\vec{B} = \mu_e \left(\vec{H} \times \vec{M}\right) \tag{7}$$

The magnitude of temperature with magnetic field is given by

$$\vec{M} = \frac{\vec{H}}{H} M\left(H, T\right) \tag{8}$$

The magnetic equation of the fundamental state is linear which is as follows:

$$M = M_0 + \chi_1 \left(H - H_0\right) - K_2 \left(T - T_a\right) \tag{9}$$

where $\chi_1 = \left(\frac{\partial M}{\partial H}\right)_{H_0, T_a}$ – Magnetic susceptibility, $K_2 = -\left(\frac{\partial M}{\partial T}\right)_{H_0, T_a}$ – Pyro magnetic coefficient, $H = \left|\vec{H}\right|$, $M = \left|\vec{M}\right|$, $M_0 = M\left(H_0, T_a\right)$, T_a – Average temperature given by $T_a = \frac{\left(T_0 + T_1\right)}{2}$.

3. Basic State of Problem

The basic state is given by $\vec{q} = \vec{q}_b\left(0,0,0\right)$, $\vec{v} = \vec{v}_b\left(0,0,0\right)$, $\rho = \rho_b\left(z\right)$, $P = P_b\left(z\right)$, $\vec{H} = \vec{H}_b\left(z\right)$, $\vec{M} = \vec{M}_b\left(z\right)$ and $\vec{B} = \vec{B}_b$.

From equation (1) to (9), we have

$$\frac{dP_b}{dz}+\rho_b g-\nabla\left(\vec{H}_b.\vec{B}_b\right)=0,\ T=T_b\left(z\right)=T_a-\beta z \tag{10}$$

where $\beta=\frac{\left(T_1-T_0\right)}{d}$, $\vec{H}_b=\left[H_0+\frac{K_2\left(T_b-T_a\right)}{1+\chi_1}\right]\hat{e}_z$, $\vec{M}_b=\left[M_0-\frac{K_2\left(T_b-T_a\right)}{1+\chi_1}\right]\hat{e}_z$.

$$\rho=\rho_b=\rho_0\left(1+\alpha\beta z\right) \tag{11}$$

$$\vec{M}_b=\frac{\vec{H}_b}{H_b}M_b \text{ and } M_b=M_0+\chi_1\left(H_b-H_0\right)-K_2\left(T_b-T_a\right) \tag{12}$$

4. Linearized Perturbation Equation

We have

$$\nabla.\vec{q}\,'=0 \tag{13}$$

$$\begin{aligned}\frac{\rho_0}{\in}\frac{\partial\vec{q}\,'}{\partial t}=&-\nabla P'+\rho_0\alpha\theta g\,\hat{e}_z-\frac{1}{k_1}\left(\mu+\varsigma\right)\vec{q}\,'+2\varsigma\left(\nabla\times\vec{v}\,'\right)+\frac{2\rho_0}{\in}\left(\vec{q}\,'\times\Omega\right)\\&+\mu_e\left(M_0+H_0\right)\nabla\frac{\partial\Phi'}{\partial z}-\mu_e K_2\beta\frac{\partial\Phi'}{\partial z}\hat{e}_z+\frac{\mu_e {K_2}^2\beta\theta\hat{e}_z}{1+\chi_1}\end{aligned} \tag{14}$$

$$\rho_0 J\frac{\partial\vec{v}\,'}{\partial t}=\varsigma\left[\frac{1}{\in}\nabla\times\vec{q}\,'-2\vec{v}\,'\right]+\left(\alpha'+\beta'\right)\nabla\left(\nabla.\vec{v}\,'\right)+\gamma'\nabla^2\vec{v}\,'+\mu_e\left(\vec{M}_b\times\vec{H}\,'+\vec{M}\,'\times\vec{H}_b\right) \tag{15}$$

$$\left[E+N_1\right]\frac{\partial\theta}{\partial t}-N_2\beta\frac{\partial}{\partial t}\left(\frac{\partial\Phi'}{\partial z}\right)=k_T\nabla^2\theta+\frac{\delta}{\rho_0 C_{v,H}}\left(\nabla\times\vec{v}\,'\right)_z\beta+\left[C_2-N_3\right]\beta\left(\vec{q}\,'\right)_z \tag{16}$$

$$\rho'=-\rho_0\alpha\theta \tag{17}$$

$$\begin{bmatrix}H_1'+M_1'=\left(1+\frac{M_0}{H_0}\right)H_1'\\H_2'+M_2'=\left(1+\frac{M_0}{H_0}\right)H_2'\\H_3'+M_3'=\left(1+\chi_1\right)H_3'-K_2\theta\\\end{bmatrix} \tag{18}$$

where $E = \in + \frac{(1-\in)\rho_s C_s}{\rho_0 C_{v,H}}$, $N_1 = \frac{\in \mu_e K_2 H_0}{\rho_0 C_{v,H}}$, $N_2 = \frac{\mu_e T_0 K_2}{\beta \rho_0 C_{v,H}}$, $N_3 = \frac{\mu_e K_2^2 T_0}{\rho_0 C_{v,H}}$,

$k_T = \frac{\chi}{\rho_0 C_{v,H}}$, $C_2 = \in + \frac{\in \mu_e K_2 H_0}{\rho_0 C_{v,H}}$, $\vec{H}' = \nabla \Phi'$, $\Phi' -$ Perturbed magnetic potential.

Converting equations (13) to (17) by the transform axon's $\vec{q}' = \frac{k_T}{d}\vec{q}^*, P' = \frac{\mu k_T}{d^2} P^*$,

$\vec{v}' = \frac{k_T}{d^2}\vec{v}^*$, $t = \frac{\rho_0 d^2}{\mu} t^*, \Omega = \frac{\mu}{\rho_0 d^2}\Omega^*, \nabla = \frac{\nabla^*}{d}, \theta = \beta d\, \theta^*, .\ z = \frac{z^*}{d}$, we have

$$\nabla . \vec{q} = 0 \tag{19}$$

$$\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = -\nabla P + R\theta \hat{e}_z - \frac{1}{K_1}(1+K)\vec{q} + 2K(\nabla \times \vec{v}) + \frac{2}{\in}(\vec{q} \times \Omega) + \frac{\mu_e d^4}{\mu k_T}(M_0 + H_0)\nabla \frac{\partial \Phi'}{\partial z} - M_2 \frac{\partial \Phi'}{\partial z}\hat{e}_z + M_1 \theta \tag{20}$$

$$\bar{J}\frac{\partial \vec{v}}{\partial t} = C_1 \nabla(\nabla . \vec{v}) - C_0 \nabla \times (\nabla \times \vec{v}) + K\left[\frac{1}{\in}\nabla \times \vec{q} - 2\vec{v}\right] + \frac{\mu_e d^4}{\mu k_T}(\vec{M}_b \times \vec{H}' + \vec{M}' \times \vec{H}_b) \tag{21}$$

$$E_r P_r \frac{\partial \theta}{\partial t} - P_r N_2 \frac{\partial}{\partial t}\left(\frac{\partial \Phi'}{\partial z}\right) = \nabla^2 \theta + \bar{\delta}(\nabla \times \vec{v}')_z + C_r(\vec{q})_z \tag{22}$$

where $R = \frac{\rho_0 g \alpha \beta d^4}{\mu k_T} -$ Thermal Rayleigh number, $P_r = \frac{\mu}{\rho_0 k_T} -$ Prandtl number,

$E_r = [E + N_1]$, $C_r = [C_2 - N_3]$,

$\bar{J} = \frac{J}{d^2}, K_1 = \frac{k_1}{d^2}, \bar{\delta} = \frac{\delta}{\rho_0 C_{v,H} d^2}, C_0 = \frac{\gamma'}{\mu d^2}, C_1 = \frac{\alpha' + \beta' + \gamma'}{\mu d^2}$, $M_1 = \frac{\mu_e K_2^2 \beta^2 d^4}{\mu k_T (1+\chi_1)}$,

$M_2 = \frac{\mu_e K_2 \beta d^4}{\mu k_T}$ $and\, W = \vec{q}.\hat{e}_z$.

5. Boundary Conditions

The boundary conditions are taken as follows:

$$W = \frac{d^2 W}{dz^2} = 0, \theta = 0\, at\, z = -\frac{d}{2}\, and\, z = \frac{d}{2}. \tag{23}$$

6. Dispersion Relation

Taking curl on both sides of equation (20), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\left(\frac{1+K}{K_1}\right)\right](\nabla\times\vec{q})=(R+M_1)\left(\frac{\partial\theta}{\partial x}\hat{e}_x+\frac{\partial\theta}{\partial x}\hat{e}_y\right)+2K\nabla\times(\nabla\times\vec{v})$$
$$+\frac{2}{\in}\nabla\times(\vec{q}\times\Omega)-M_2\left[\left(\frac{\partial}{\partial x}\hat{e}_x+\frac{\partial}{\partial x}\hat{e}_y\right)\frac{\partial\Phi'}{\partial z}\right] \tag{24}$$

Let us take $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}, \nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}, D=\frac{\partial}{\partial z},$

$\zeta_z=(\nabla\times\vec{q})_z, \Omega_z'=(\nabla\times\vec{v})_z$, $R_1=R+M_1$.

Taking curl and z-component of equation (24) and (21), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\left(\frac{1+K}{K_1}\right)\right]\nabla^2W=R_1\nabla_1^2\theta+2K\nabla^2\Omega_z'\hat{e}_z-\frac{2}{\in}\Omega D\zeta_z-M_2\nabla_1^2\frac{\partial\Phi'}{\partial z} \tag{25}$$

$$\bar{J}\frac{\partial\Omega_z'}{\partial t}=C_0\nabla^2\Omega_z'-K\left[\frac{1}{\in}\nabla^2W+2\Omega_z'\right] \tag{26}$$

Taking z-component of equation (24) and (22), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\left(\frac{1+K}{K_1}\right)\right]\zeta_z=\frac{2\Omega}{\in}DW \tag{27}$$

$$E_rP_r\frac{\partial\theta}{\partial t}-P_rN_2\frac{\partial}{\partial t}\left(\frac{\partial\Phi'}{\partial z}\right)=\nabla^2\theta+\bar{\delta}\,\Omega_z'+C_rW \tag{28}$$

From equation (18), we have

$$(1+\chi_1)\frac{\partial^2\Phi'}{\partial z^2}+\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\Phi'-K_2\frac{\partial\theta}{\partial z}=0 \tag{29}$$

6.1 Normal Mode Analysis

Let $[W,\zeta_z,\theta,\Omega_z',\Phi']=[W(z),X(z),\Theta(z),G(z),\Phi(z)]\exp.[ik_xx+ik_yy+\sigma t]$

Applying normal mode of equation (25) to (29), we have

$$\left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)\right]\left(D^2-a^2\right)W=-R_1a^2\Theta+2K\left(D^2-a^2\right)G-\frac{2}{\in}\Omega DX+M_2a^2D\Phi \quad (30)$$

$$\left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)\right]X=\frac{2\Omega}{\in}DW \quad (31)$$

$$\left[m\sigma+2A-\left(D^2-a^2\right)\right]G=-\frac{A}{\in}\left(D^2-a^2\right)W \quad (32)$$

$$\left[E_rP_r\sigma-\left(D^2-a^2\right)\right]\Theta=P_rN_2\sigma D\Phi-\bar{\delta}G+C_rW \quad (33)$$

$$D^2\Phi-M_3a^2\Phi-D\Theta=0 \quad (34)$$

where $a^2=k_x^2+k_y^2$ - wave number, $\sigma=\sigma_r+i\sigma_r$ - stability parameter, $M_3=\left(1+M_0/H_0\right)/(1+\chi_1)$ and $m=\frac{\bar{J}A}{K}$, $A=\frac{K}{C_0}$, A - ratio between the micro-polar viscous effect and micro-polar diffusion effects.

Now, the boundary conditions become

$$W=D^2W=0=X=DX=G,\Theta=0\,at\,z=-1/2\,to\,z=1/2 \quad (35)$$

$D^{2n}W=0\,at\,z=-1/2\,to\,z=1/2$, where n is a positive integer.

The proper solution satisfying (35) can be taken as $W=W_0\cos\pi z$, W_0 - being a constant.

Eliminating Θ,G,Φ,X from (30) to (34) and putting the value of $W\ and\ b=\pi^2+a^2$, we have

$$\begin{aligned}b\left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)\right]^2\left[m\sigma+2A+b\right]\left[P_rN_2\sigma\pi^2-\left(E_rP_r\sigma+b\right)\left(\pi^2+M_3a^2\right)\right]=R_1a^2\left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)\right]\\ \left(\pi^2+M_3a^2\right)\left[C_r\left(m\sigma+2A+b\right)-\frac{\bar{\delta}Ab}{\in}\right]+\frac{2KAb^2}{\in}\left[P_rN_2\sigma\pi^2-\left(E_rP_r\sigma+b\right)\left(\pi^2+M_3a^2\right)\right] \quad (36)\\ \left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)\right]-\frac{4\Omega^2\pi^2}{\in^2}\left(m\sigma+2A+b\right)\left[P_rN_2\sigma\pi^2-\left(E_rP_r\sigma+b\right)\left(\pi^2+M_3a^2\right)\right]\\ -M_2a^2\pi^2\left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)\right]\left[C_r\left(m\sigma+2A+b\right)-\frac{\bar{\delta}Ab}{\in}\right]\end{aligned}$$

7. Stationary Convection

Putting $\sigma = 0$ in equation (36), we have

$$R_1 = \frac{1}{a^2}\left[\frac{b^2(2A+b)\left(\frac{1+K}{K_1}\right) - \frac{2KAb^3}{\in}}{\left[C_r(2A+b) - \frac{\bar{\delta}Ab}{\in}\right]} + \frac{\frac{4\Omega^2\pi^2}{\in^2}b(2A+b)}{\left(\frac{1+K}{K_1}\right)\left[C_r(2A+b) - \frac{\bar{\delta}Ab}{\in}\right]} + \frac{M_2a^2\pi^2}{\left(\pi^2 + M_3a^2\right)}\right] \quad (37)$$

To discuss the behavior of permeability, rotation, coupling parameter, micro-polar coefficient, micro-polar heat transfer parameter and find the nature of $\frac{dR_1}{dK_1}, \frac{dR_1}{d\Omega}, \frac{dR_1}{dK}, \frac{dR_1}{dA}$ *and* $\frac{dR_1}{d\bar{\delta}}$ respectively, we write

$$\frac{dR_1}{dK_1} = -\frac{b(2A+b)}{a^2(1+K)\left[C_r(2A+b) - \bar{\delta}Ab\in^{-1}\right]}\left[b\left(\frac{1+K}{K_1}\right)^2 - \frac{4\Omega^2\pi^2}{\in^2}\right] \quad (38)$$

$$\frac{dR_1}{dK_1} < 0 \; if \; K_1 < \frac{\in(1+K)\sqrt{b}}{2\pi\Omega} \; and \; \bar{\delta} < \frac{\in C_r}{A}$$

From equation (38), we can say that the medium permeability has destabilizing effect when $K_1 < \frac{\in(1+K)\sqrt{b}}{2\pi\Omega}$ *and* $\bar{\delta} < \frac{\in C_r}{A}$.

$$\frac{dR_1}{d\Omega} = \frac{8\Omega\pi^2 \in^{-2} b(2A+b)}{a^2\left(\frac{1+K}{K_1}\right)\left[C_r(2A+b) - \frac{\bar{\delta}Ab}{\in}\right]} \quad (39)$$

$$\frac{dR_1}{d\Omega} > 0 \; if \; \bar{\delta} < \frac{\in C_r}{A}$$

From equation (39), we can say that the rotation has stabilizing effect when the $\bar{\delta} < \frac{\in C_r}{A}$.

$$\frac{dR_1}{dK} = \frac{b}{a^2\left(\frac{1+K}{K}\right)^2\left[C_r(2A+b) - \frac{\bar{\delta}Ab}{\in}\right]}\left[\begin{array}{c} 2Ab\left(\frac{1}{K_1} - \frac{b}{\in}\right)\left(\frac{1+K}{K_1}\right)^2 + \frac{1}{K_1} \\ \left\{b^2\left(\frac{1+K}{K_1}\right)^2 - \frac{4\Omega^2\pi^2(2A+b)}{\in^2}\right\} \end{array}\right] \quad (40)$$

$$\frac{dR_1}{dK} > 0 \, if \; K > \frac{2\Omega\pi K_1\sqrt{(2A+b)}}{\in b}, \frac{1}{K_1} > \frac{b}{\in} and \; \bar{\delta} < \frac{\in C_r}{A}$$

From equation (40), we can say that the coupling parameter has stabilizing effect when the $K > \frac{2\Omega\pi K_1\sqrt{(2A+b)}}{\in b}, \frac{1}{K_1} > \frac{b}{\in} and \; \bar{\delta} < \frac{\in C_r}{A}$.

$$\frac{dR_1}{dA} = \frac{\frac{b^4}{\in}\left(\frac{1+K}{K_1}\right)^2\left[\bar{\delta} - 2C_rK_1 + \frac{2C_rK_1}{(1+K)}\right] + \frac{4\Omega^2\pi^2\bar{\delta}b^3}{\in^3}}{a^2\left(\frac{1+K}{K_1}\right)\left[C_r(2A+b) - \frac{\bar{\delta}Ab}{\in}\right]^2} \tag{41}$$

$$\frac{dR_1}{dA} > 0 \, if \; \bar{\delta} > 2C_rK_1$$

From equation (41), we can say that the micro-polar coefficient has stabilizing effect when the $\bar{\delta} < 2C_rK_1$.

$$\frac{dR_1}{d\bar{\delta}} = \frac{\frac{Ab}{\in}}{a^2\left[C_r(2A+b) - \frac{\bar{\delta}Ab}{\in}\right]^2}\left[\begin{array}{c} b^3\left\{\left(\frac{1+K}{K_1}\right) - \frac{2KA}{\in}\right\} + 2Ab^2\left(\frac{1+K}{K_1}\right) \\ + \frac{4\Omega^2\pi^2b(2A+b)}{\in^2}\left(\frac{K_1}{1+K}\right) \end{array}\right] \tag{42}$$

$$\frac{dR_1}{d\bar{\delta}} > 0 \, if \; \frac{1}{K_1} > \frac{A}{\in}$$

From equation (42), we can say that the micro-polar heat transfer parameter has stabilizing effect when the $\frac{1}{K_1} > \frac{A}{\in}$.

8. Oscillatory Convection

Putting $\sigma = i\sigma_i$ in equation (36), we get real and imaginary parts and eliminating R_1 between them, we have

$$f_0\sigma_i^4 + f_1\sigma_i^2 + f_2 = 0$$

Putting $s = \sigma_i^2$, we have

$$f_0 s^2 + f_1 s + f_2 = 0 \tag{43}$$

where $f_0 = a_1 q_1 - p_1 b_1$, $f_1 = a_2 q_1 - p_2 b_1 - p_1 b_2$, $f_2 = a_3 q_1 - p_2 b_2$,

$$a_1 = b\left[\frac{1}{\in^2}\left\{P_r N_2 m\pi^2 - E_r P_r m\left(\pi^2 + a^2 M_3\right)\right\}\right],$$

$$b_1 = -a^2\left(\pi^2 + M_3 a^2\right)\frac{mC_r}{\in},\ b_2 = a^2\left(\pi^2 + M_3 a^2\right)\left[\left(\frac{1+K}{K_1}\right)\left\{C_r\left(2A+b\right) - \frac{\bar{\delta} Ab}{\in}\right\}\right],$$

$$a_2 = b\left[\begin{array}{c} \frac{(2A+b)b}{\in^2}\left(\pi^2 + M_3 a^2\right) + \frac{2}{\in}\left(\frac{1+K}{K_1}\right)\left\{\begin{array}{c} mb\left(\pi^2 + M_3 a^2\right) + \left(2A+b\right) \\ \left(\pi^2 + M_3 a^2\right) E_r P_r - \left(2A+b\right) P_r N_2 \pi^2 \end{array}\right\} \\ + \left(\frac{1+K}{K_1}\right)^2 \left\{E_r P_r m\left(\pi^2 + a^2 M_3\right) - P_r N_2 m\pi^2\right\} \end{array}\right] - \frac{2KA}{\in^2},$$

$$b^2\left[E_r P_r\left(\pi^2 + M_3 a^2\right) - P_r N_2 \pi^2\right] - \frac{4\Omega^2\pi^2}{\in^2}\left[E_r P_r m\left(\pi^2 + a^2 M_3\right) - P_r N_2 m\pi^2\right] - M_2 a^2 \pi^2 \frac{mC_r}{\in}$$

$$a_3 = b^2\left(2A+b\right)\left(\frac{1+K}{K_1}\right)^2\left\{E_r P_r m\left(\pi^2 + a^2 M_3\right) - P_r N_2 m\pi^2\right\} + \frac{2KAb^3}{\in}\left(\pi^2 + M_3 a^2\right)\left(\frac{1+K}{K_1}\right),$$
$$-\frac{4\Omega^2\pi^2}{\in^2}\left(2A+b\right)b\left(\pi^2 + M_3 a^2\right) + M_2 a^2\pi^2\left(\frac{1+K}{K_1}\right)\left[C_r\left(2A+b\right) - \frac{\bar{\delta} Ab}{\in}\right]$$

$$q_1 = a^2\left(\pi^2 + M_3 a^2\right)\left[\frac{1}{\in}\left\{C_r\left(2A+b\right) - \frac{\bar{\delta} Ab}{\in}\right\} + mC_r\left(\frac{1+K}{K_1}\right)\right],$$

$$p_1 = b\left[\begin{array}{c} \frac{1}{\in^2}\left\{mb\left(\pi^2 + M_3 a^2\right) + \left(2A+b\right)\left(\pi^2 + M_3 a^2\right) E_r P_r - \left(2A+b\right) P_r N_2 \pi^2\right\} \\ + \frac{2}{\in}\left(\frac{1+K}{K_1}\right)\left\{E_r P_r m\left(\pi^2 + a^2 M_3\right) - P_r N_2 m\pi^2\right\} \end{array}\right]$$
$$+ \frac{2KAb^2}{\in}\left[\frac{1}{\in}\left(\pi^2 + M_3 a^2\right) + \left(\frac{1+K}{K_1}\right)\left(\pi^2 + M_3 a^2\right) E_r P_r - \left(\frac{1+K}{K_1}\right) P_r N_2 \pi^2\right],$$
$$+ \frac{4\Omega^2\pi^2}{\in^2}\left[mb\left(\pi^2 + a^2 M_3\right) - \left(2A+b\right)\left(\pi^2 + M_3 a^2\right) E_r P_r - \left(2A+b\right) P_r N_2 \pi^2\right]$$
$$+ M_2 a^2\pi^2\left[\frac{1}{\in}\left\{C_r\left(2A+b\right) - \frac{\bar{\delta} Ab}{\in}\right\} + mC_r\left(\frac{1+K}{K_1}\right)\right]$$

$$p_2 = -b\left[\frac{2(2A+b)b}{\in}\left(\frac{1+K}{K_1}\right)\left(\pi^2+M_3a^2\right)+\left(\frac{1+K}{K_1}\right)^2\left\{\begin{matrix} mb\left(\pi^2+M_3a^2\right)+(2A+b) \\ \left(\pi^2+M_3a^2\right)E_rP_r-(2A+b)P_rN_2\pi^2 \end{matrix}\right\}\right]$$
$$-\frac{2KAb^2}{\in}\left[\frac{1}{\in}\left(\pi^2+M_3a^2\right)+\left(\frac{1+K}{K_1}\right)\left(\pi^2+M_3a^2\right)E_rP_r-\left(\frac{1+K}{K_1}\right)P_rN_2\pi^2\right] .$$
$$+\frac{4\Omega^2\pi^2}{\in^2}\left[mb\left(\pi^2+a^2M_3\right)-(2A+b)\left(\pi^2+M_3a^2\right)E_rP_r-(2A+b)P_rN_2\pi^2\right]$$
$$+M_2a^2\pi^2\left[\frac{1}{\in}\left\{C_r(2A+b)-\frac{\bar{\delta}Ab}{\in}\right\}+mC_r\left(\frac{1+K}{K_1}\right)\right]$$

From equation (43), we say that $s=\sigma_i^2$ is positive. The sum of roots of equation (43) is positive and it is impossible if $f_0>0$ and $f_1>0$, the sum of roots of equation (43) is $-f_1/f_0$.

Now $f_0>0$ and $f_1>0$ are the sufficient conditions for the non-existence of over stability.

If $f_0>0$ and $f_1>0$, then $\bar{\delta}<\frac{\in}{A}, KE_rP_r<2 \text{ and } AKb^2<2m\pi^2\Omega^2$.

9. Numerical Calculation

We show the graphical effect of different parameters from equation (36) as follows:

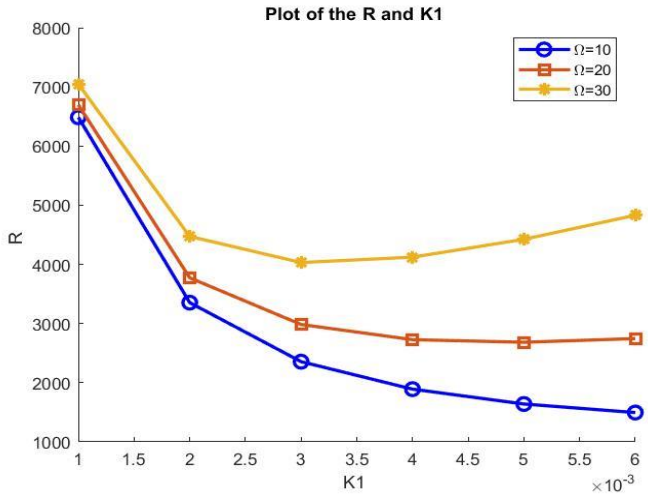


Fig. 1 : $E_r=1, P_r=2, \in=0.5, A=0.1, K=0.2, \Omega=10 \text{ and } \bar{\delta}=0.05$.

Fig.1 shows the variation of Rayleigh number R_1 with respect to medium permeability K_1 i.e. medium permeability increases when Rayleigh number decreases.

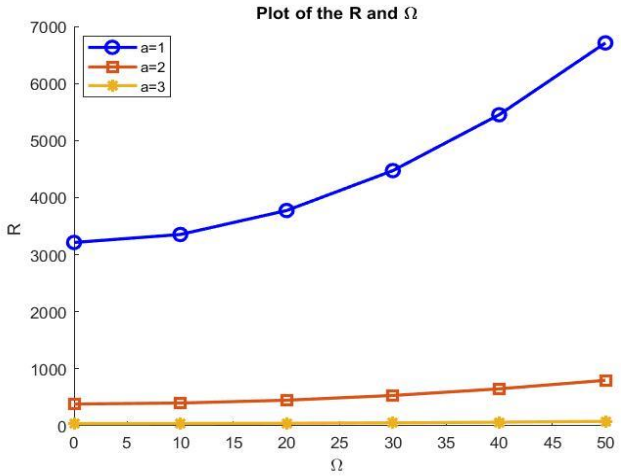


Fig. 2 : $E_r = 1, P_r = 2, \in = 0.5,\ A = 0.1, K = 0.2, K_1 = 0.002, \text{and} \overline{\delta} = 0.05.$

Fig.2 represent the plot of Rayleigh number R_1 versus rotation Ω i.e. rotation increases when the Rayleigh number increases.

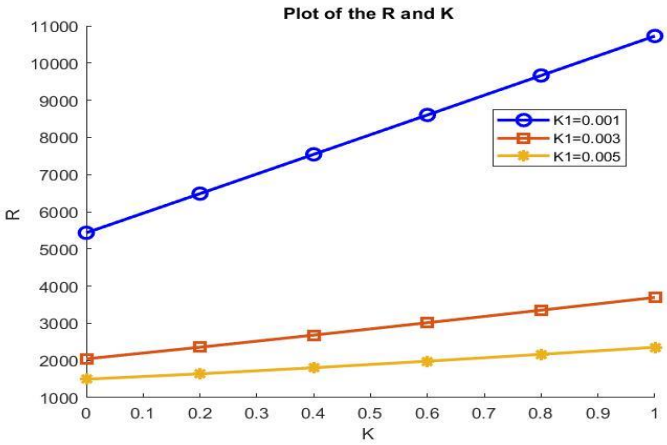


Fig.3 : $E_r = 1, P_r = 2, \in = 0.5,\ A = 0.1, \Omega = 10, K_1 = 0.002, \text{and} \overline{\delta} = 0.05.$

Fig.3 shows the variation of Rayleigh number R_1 with respect to coupling parameter K i.e. coupling parameter K increases when the Rayleigh number increases.

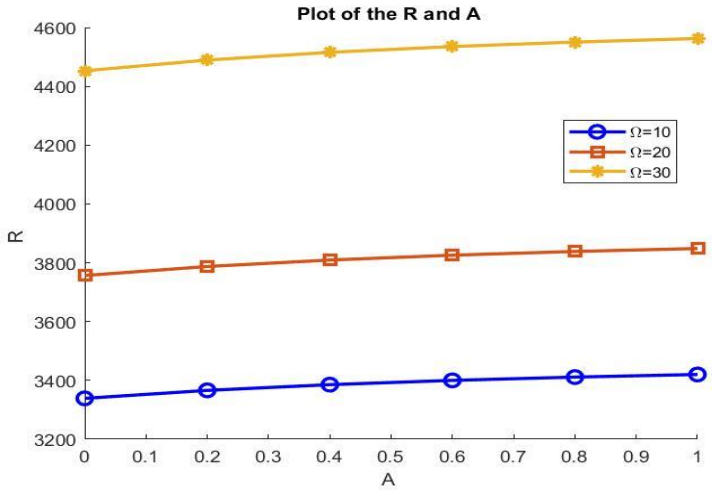


Fig.4 : $E_r = 1, P_r = 2, \in = 0.5, \Omega = 10, K = 0.2, K_1 = 0.002, \text{and} \bar{\delta} = 0.05.$

Fig.4 represent the plot of Rayleigh number R_1 versus micro-polar coefficient A i.e. micro-polar coefficient increases when the Rayleigh number increases.

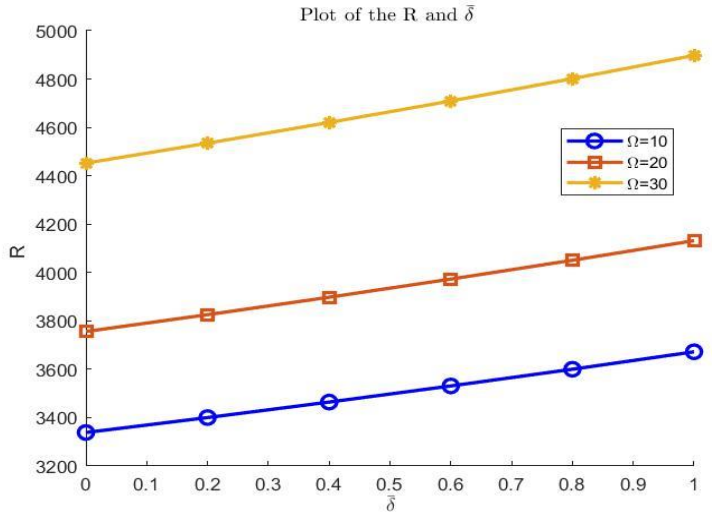


Fig.5: $E_r = 1, P_r = 2, \in = 0.5, A = 0.1, K = 0.2, K_1 = 0.002, \text{and} \Omega = 10.$

Fig.5 plot between Rayleigh number R_1 and micro-polar heat conduction parameter $\bar{\delta}$ i.e. heat conduction parameter increases when the Rayleigh number increases.

10. Conclusions

We have the following conclusions:

10.1 For Stationary Convection

If $\frac{dR_1}{dK_1} < 0$, then the effect of medium permeability is destabilizing.

If $\frac{dR_1}{d\Omega} > 0$, then the effect of rotation is stabilizing.

If $\frac{dR_1}{dK} > 0$, then the effect of coupling parameter is stabilizing.

If $\frac{dR_1}{dA} > 0$, then the effect of micro-polar coefficient is stabilizing.

If $\frac{dR_1}{d\bar{\delta}} > 0$, then the effect of micro-polar heat conduction is stabilizing.

10.2 Oscillatory Convection

The sufficient condition for the non-existence of over stability is $\bar{\delta} < \frac{\in}{A}, KE_r P_r < 2$ and $AKb^2 < 2m\pi^2\Omega^2$.

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