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EFFECT OF THE COUPLE-STRESS ON MICRO POLAR FLUID FLOW SATURATING A POROUS MEDIUM

DEVILAL KUMAWAT, VIJAY MEHTA and VIKAS TAILOR ✉

Abstract

Effect of the couple-stress on micro polar fluid layer heated from below in a porous medium is studied. The dispersion relation has been analyzed using normal mode and it is found that the medium permeability has destabilizing effect. The couple-stress parameter, coupling parameter, heat conduction parameter and micro-polar coefficient have stabilizing effect. The sufficient condition for the non-existence of over stability has been obtained.

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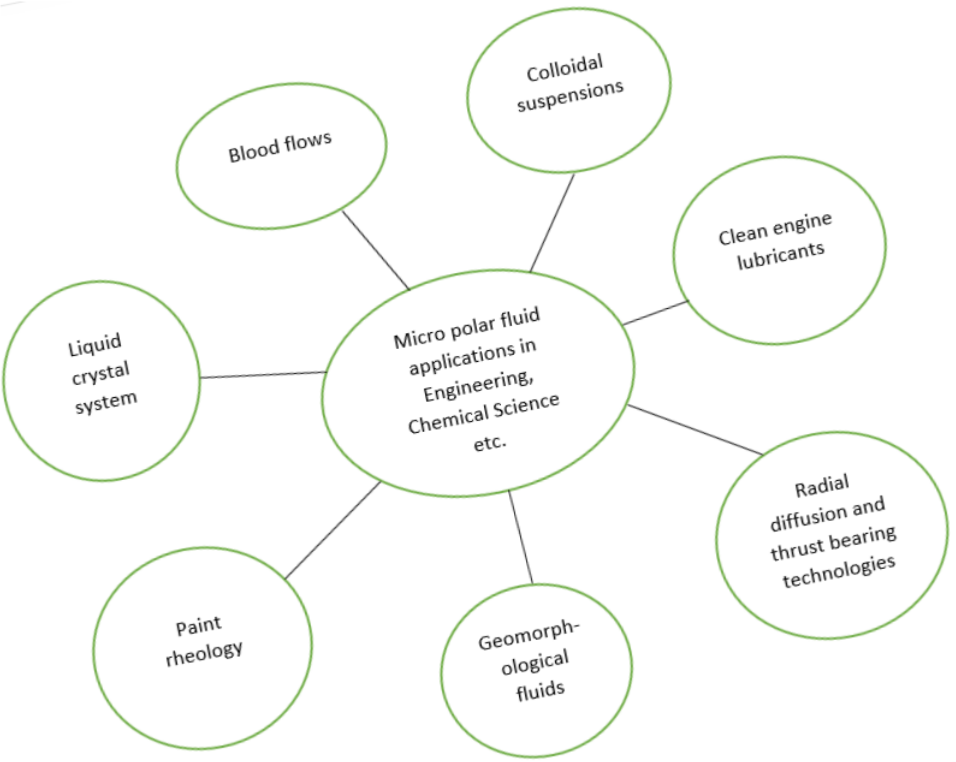
1. Introduction

The general theory of micro polar fluid was introduced Eringen [4, 5]. The theory of micro polar fluid and viscous fluid in a vertical channal, developed by chamkha et al. [3] and Kumar [6]. Colloidal fluids, animal blood, liquid crystals, polymeric suspension are few examples of micro polar fluids [5, 9]. Sharma and Gupta [12] investigated the thermal convection on micro polar fluid in porous medium. Chamkha [2] discussed the problem of hydro magnetic two-phase flow in a channel. Sunil et al. [16] analyzed rotation and different parameters on a micro-polar ferromagnetic fluid flow. Sharma et al. [13] studied the effect of porosity, magnetic field and electrically conducting. Wooding [17] discussed the Rayleigh instability of flow through a porous medium. Mittal and Rana [10] investigated the medium permeability, suspended particles and other parameters on the micro-polar ferromagnetic fluid flow saturating a porous medium.

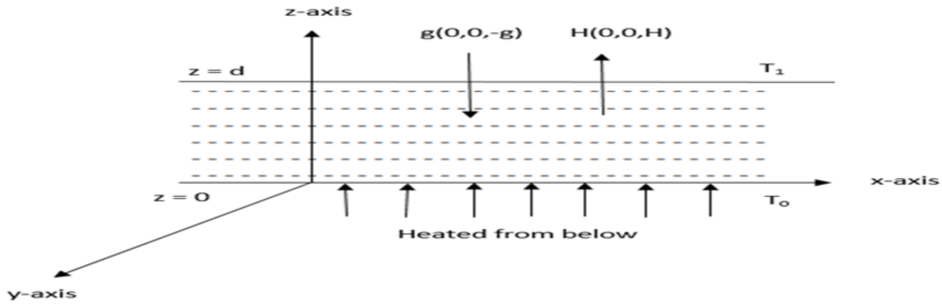
Stokes [15] studied the classical theory of couple-stress fluid. Kumawat and mehta [7] discussed the effect rotation and other parameters on the micro polar ferromagnetic fluid flow. Banyal and Singh [1] investigated the rotation on the couple-stress fluid in a porous medium. Shivakumara et al. [14] analyzed the onset of convection in a couple-stress fluid flow saturating a porous medium by the Galerkin method. Kumawat et al. [8] studied the effect of the couple-stress on micro polar rotating fluid flow. Xiong et al. [18] investigated the couple stress on couple stress fluid flow between parallel plates with thermal convection. Pundir et al. [11] discussed the effect of medium permeability, couple-stress parameter and magnetization on ferromagnetic fluid layer heated from below in a porous medium with hall current. We have highlighted some applications of this theory.



In this paper, we attempt to study the couple-stress on micro-polar fluid flow saturating a porous medium. To our knowledge this problem has not yet been investigated using the generalized Darcy's model.

2. Mathematical Formulation

An infinite, horizontal, incompressible micro-polar fluid layer of thickness d is assumed and has porosity $\in$ and medium permeability k_1 . The upper limit $z = d$ and lower limit $z = 0$ are maintained at constant but varying temperatures T_0 and T_1 such that a study adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ has been continued.



The equation of continuity, motion, angular momentum, temperature and basic's state are

$$\nabla.\vec{q} = 0, \tag{2.1}$$

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right] = -\nabla P - \rho g \hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q} - \frac{1}{k_1}(\mu + \varsigma)\vec{q} + \varsigma(\nabla \times \vec{v}), \tag{2.2}$$

$$\rho_0 J\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\in}(\vec{v}.\nabla)\vec{v}\right] = (\alpha' + \beta')\nabla(\nabla.\vec{v}) + \gamma'\nabla^2\vec{v} + \frac{\varsigma}{\in}(\nabla \times \vec{q}) - 2\varsigma\vec{v}, \tag{2.3}$$

$$[\in \rho_0 C_v + (1- \in)\rho_s C_s]\frac{\partial T}{\partial t} + \rho_0 C_v(\vec{q}.\nabla)T = \chi\nabla^2 T + \delta(\nabla \times \vec{v}).\nabla T, \tag{2.4}$$

$$\rho = \rho_0[1 - \alpha(T - T_a)]. \tag{2.5}$$

where, ρ - Fluid density, ρ_0 - Reference density, $\vec{q}$ - Filter velocity, $\vec{v}$ - Spin (micro rotation), μ - Shear kinematic viscosity coefficient, ς - Coupling viscosity coefficient, P - Pressure, μ' - Couple stress viscosity, $\hat{e}_z$ - Unit vector in z-direction, α' - Bulk spin viscosity coefficient, β' - Shear spin viscosity coefficient, γ' - Micro-polar viscosity coefficient, J - Micro inertia constant, t - time, C_v - Specific heat at constant volume and magnetic field, C_s - Specific heat of solid (Porous Material Matrix), ρ_s - Density of solid matrix, χ - Thermal conductivity, T - Temperature, δ - Micro-polar heat conduction coefficient, α - Coefficient of thermal expansion.

3. Basic State

The basic's state is

$$\vec{q} = \vec{q}_b(0,0,0), \ \vec{v} = \vec{v}_b(0,0,0), \ \rho = \rho_b(z), P = P_b(z).$$

From equation (2.1) to (2.5)

$$\frac{dP_b}{dz} + \rho_b g = 0, \tag{3.1}$$

$$T = T_b(z) = -\beta z + T_a\,;\; where\,\beta = \frac{(T_1 - T_0)}{d}, \tag{3.2}$$

$$\rho_b = \rho_0(1 + \alpha\beta z). \tag{3.3}$$

4. Linearize Perturbation Equations

$$\nabla.\vec{q}' = 0, \tag{4.1}$$

$$\frac{\rho_0}{\in}\frac{\partial \vec{q}'}{\partial t} = -\nabla P' + \alpha\theta g\hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q}' - \frac{1}{k_1}(\mu + \varsigma)\vec{q}' + \varsigma(\nabla\times\vec{v}'), \tag{4.2}$$

$$\rho_0 J\frac{\partial \vec{v}'}{\partial t} = (\alpha' + \beta')\nabla(\nabla.\vec{v}') + \gamma'\nabla^2\vec{v}' + \frac{\varsigma}{\in}(\nabla\times\vec{q}') - 2\varsigma\vec{v}', \tag{4.3}$$

$$E\frac{\partial\theta}{\partial t} + (\vec{q}.\nabla)T_b = k_T\nabla^2\theta - \frac{\delta}{\rho_0 C_v}(\nabla\times\vec{v}')_z\beta + \beta(\vec{q}')_z, \tag{4.4}$$

$$\rho' = -\rho_0\alpha\theta. \tag{4.5}$$

Converting equation (4.1) to (4.5) by the following transform $\vec{q}' = \frac{k_T}{d}\vec{q}*$, $P' = \frac{\mu k_T}{d^2}P*$, $\vec{v}' = \frac{k_T}{d^2}\vec{v}*$, $t = \frac{\rho_0 d^2}{\mu}t*$, $\nabla = \frac{\nabla*}{d}$, $\theta = \beta d\,\theta*$, we have

$$\nabla.\vec{q} = 0, \tag{4.6}$$

$$\frac{1}{\in}\frac{\partial\vec{q}}{\partial t} = -\nabla P + R\theta\hat{e}_z + \left(1 - F\nabla^2\right)\nabla^2\vec{q} - \frac{1}{K_1}(1 + K)\vec{q} + K(\nabla\times\vec{v}), \tag{4.7}$$

$$\bar{J}\frac{\partial\vec{v}'}{\partial t} = C_1\nabla(\nabla.\vec{v}) - C_0\nabla(\nabla\times\vec{v}) + K\left\{\frac{1}{\in}(\nabla\times\vec{q}) - 2\vec{v}\right\}, \tag{4.8}$$

$$EP_r\frac{\partial\theta}{\partial t} = \nabla^2\theta - \bar{\delta}(\nabla\times\vec{v})_z + (\vec{q})_z. \tag{4.9}$$

where, $R = \frac{\rho_0 g\alpha\beta d^4}{\mu k_T}$ - Thermal Rayleigh number, $P_r = \frac{\mu}{\rho_0 k_T}$ - Prandtl number, $F = \frac{\mu'}{\rho_0 d^2}$, $E = \in + \frac{(1-\in)\rho_s C_s}{\rho_0 C_v}$, $\bar{J} = \frac{J}{d^2}$, $K_1 = \frac{k_1}{d^2}$, $\bar{\delta} = \frac{\delta}{\rho_0 C_v d^2}$, $C_0 = \frac{\gamma'}{\mu d^2}$, $C_1 = \frac{\alpha'+\beta'+\gamma'}{\mu d^2}$ and $W = \vec{q}.\hat{e}_z$.

5. Boundary conditions

The boundary condition is

$$W = \frac{d^2W}{dz^2} = 0,\; \theta = 0\; at\, z = 0\; and\, z = d. \tag{5.1}$$

6. Dispersion Relation

Taking curl on both side equation (4.7), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\left(\frac{1+K}{K_1}\right)-\left(1-F\nabla^2\right)\nabla^2\right](\nabla\times\vec{q})=R\left(\frac{\partial\theta}{\partial x}\hat{e}_x+\frac{\partial\theta}{\partial x}\hat{e}_y\right)$$
$$+K\nabla\times(\nabla\times\vec{v})\,. \quad (6.1)$$

Assume $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$, $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, $D=\frac{\partial}{\partial z}$, $\zeta_z=(\nabla\times\vec{q})_z$, $\Omega_z{}'=(\nabla\times\vec{v})_z$. Taking curl and z-component of equations (6.1) and (4.8), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\left(\frac{1+K}{K_1}\right)-\left(1-F\nabla^2\right)\nabla^2\right]\nabla^2W=R\nabla_1^2\theta+K\nabla^2\,\Omega_z{}'\,\hat{e}_z, \quad (6.2)$$

$$\bar{J}\frac{\partial\Omega_z{}'}{\partial t}=C_0\nabla^2\Omega_z{}'\,-K\left[\frac{1}{\in}\nabla^2W+2\Omega_z{}'\right]. \quad (6.3)$$

Taking z-component of equation (4.9), we have

$$EP_r\frac{\partial\theta}{\partial t}=\nabla^2\theta-\bar{\delta}\,\Omega_z{}'\,+W. \quad (6.4)$$

6.1 Normal Mode Analysis

Let $[W,\,\zeta_z,\,\theta,\,\Omega_z{}']=[W(z)\,,\,X(z)\,,\,\Theta(z)\,,\,G(z)]\ \exp.\left[i\,k_xx+i\,k_yy+\sigma t\right]$
Apply normal mode of equation (6.2) to (6.4), then

$$\left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W=-Ra^2\Theta$$
$$+K\left(D^2-a^2\right)G, \quad (6.5)$$

$$\left[m\sigma+2A-\left(D^2-a^2\right)\right]G=-\frac{A}{\in}\left(D^2-a^2\right)W, \quad (6.6)$$

$$\left[EP_r\sigma-\left(D^2-a^2\right)\right]\Theta=-\bar{\delta}G+W. \quad (6.7)$$

where, $a^2=k_x^2+k_y^2$ - wave number, $\sigma=\sigma_r+i\,\sigma_r$ - stability parameter and $m=\frac{\bar{J}A}{K}$, $A=\frac{K}{C_0}$, A - ratio between the micro-polar viscous effect and micro-polar diffusion effects.

$$W=D^2W=0=X=DX=G,\Theta=0\,at\,z=0\;to\;z=1 \quad (6.8)$$

$$D^{2n}W=0\;at\;z=0\;to\;z=1,\,n>0.$$

Thus, the proper solution of equation (6.8) is

$$W=W_0\,\sin\pi z\,.$$

Eliminating Θ, G *and* X from (6.5) to (6.7) and putting the value of W *and* $b = \pi^2+a^2$, we have

$$b\left[\left\{\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)+Fb^2+b\right\}\right][m\sigma+2A+b]\left[EP_r\sigma+b\right]=Ra^2$$
$$\left[(m\sigma+2A+b)-\frac{\bar{\delta}Ab}{\in}\right]+\frac{KAb^2}{\in}\left[EP_r\sigma+b\right]. \quad (6.9)$$

Now, the absence of couple-stress in equation (6.9), we have

$$b\left[\left\{\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)\right\}\right][m\sigma+2A+b]\left[EP_r\sigma+b\right]=Ra^2$$
$$\left[(m\sigma+2A+b)-\frac{\bar{\delta}Ab}{\in}\right]+\frac{KAb^2}{\in}\left[EP_r\sigma+b\right].$$

This result derived by Sharma and Gupta (1995).

7. Stationary Convection

Putting $\rho=0$ in equation (6.9), we have

$$R=\frac{1}{a^2}\left[\frac{b^2(2A+b)\left(\frac{1+K}{K_1}+Fb^2+b\right)-\frac{KAb^3}{\in}}{\left((2A+b)-\frac{\bar{\delta}Ab}{\in}\right)}\right]. \quad (7.1)$$

Now, the absence of couple-stress in equation (7.1), we have

$$R=\frac{1}{a^2}\left[\frac{b^3\left\{\left(\frac{1+K}{K_1}\right)-\frac{KA}{\in}\right\}+2Ab^2\left(\frac{1+K}{K_1}\right)}{\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)}\right].$$

It is derived by Sharma and Gupta (1995).

To discussed the behavior of permeability,Couple-stress parameter, coupling parameter, micro-polar coefficient, micro-polar heat transfer parameter and find the nature of $\frac{dR}{dK_1}$, $\frac{dR}{dF}$, $\frac{dR}{dK}$, $\frac{dR}{dA}$ *and* $\frac{dR}{d\bar{\delta}}$ respectively, then

$$\frac{dR}{dK_1}=\frac{-b^2(2A+b)(1+K)}{a^2{K_1}^2\left((2A+b)-\frac{\bar{\delta}Ab}{\in}\right)}, \quad (7.2)$$

$$\frac{dR}{dK_1}<0 \; if \; \bar{\delta}<\frac{\in}{A}.$$

From equation (7.2), we can say that the permeability has destabilizing effect.

$$\frac{dR}{dF}=\frac{b^4(2A+b)}{a^2\left((2A+b)-\frac{\bar{\delta}Ab}{\in}\right)}, \quad (7.3)$$

$$\frac{dR}{dF}>0 \; if \; \bar{\delta}<\frac{\in}{A}.$$

From equation (7.3), we can say that the couple-stress parameter has stabilizing effect.

$$\frac{dR}{dK} = \frac{\left[Ab^2\left(\frac{2}{K_1} - \frac{b}{\in}\right) + \frac{b^3}{K_1}\right]}{a^2\left((2A + b) - \frac{\bar{\delta}Ab}{\in}\right)}, \tag{7.4}$$

$$\frac{dR}{dK} > 0 \ if \ \frac{2}{K} > \frac{b}{\in} \ and \ \bar{\delta} < \frac{\in}{A}.$$

From equation (7.4), we can say that the coupling parameter has stabilizing effect.

$$\frac{dR}{dA} = \frac{\frac{b^4}{\in}\left[\bar{\delta}\left(\frac{1+K}{K_1} + Fb^2 + b\right) - K\right]}{a^2\left((2A + b) - \frac{\bar{\delta}Ab}{\in}\right)^2}, \tag{7.5}$$

$$\frac{dR}{dA} > 0 \ \ if \ \bar{\delta}\left(\frac{1+K}{K_1} + Fb^2 + b\right) > K.$$

From equation (7.5), we can say that the micro-polar coefficient has stabilizing effect.

$$\frac{dR}{d\bar{\delta}} = \frac{Ab}{\in}\frac{\left[\left(2Ab^3 - \frac{KAb^3}{\in}\right) + b^4 + (2A + b)\left(\frac{1+K}{K_1} + Fb^2\right)\right]}{a^2\left((2A + b) - \frac{\bar{\delta}Ab}{\in}\right)^2}, \tag{7.6}$$

$$\frac{dR}{d\bar{\delta}} > 0 \, if \, K < 2 \in.$$

From equation (7.6), we can say that the micro-polar heat transfer parameter has stabilizing effect.

8. Oscillatory Convection

Putting $\sigma = i\sigma_i$ in equation (6.9) then we get real and imaginary part, eliminating R between them, we have

$$f_0\sigma_i^2 + f_1 = 0. \tag{8.1}$$

where,

$$f_0 = a_1q_1 - p_1b_1,$$
$$f_1 = a_2q_1 - p_2b_1,$$

$$P_1 = -\frac{EP_rmb}{\in}, \quad q_1 = ma^2, \quad b_1 = a^2\left[(2A + b) - \frac{\bar{\delta}Ab}{\in}\right],$$

$$a_1 = -\left[\left\{\left(\frac{1+K}{K_1} + Fb^2 + b\right) + \right\}E_rP_rmb + \frac{b}{\in}\{(2A + b)\,EP_r + mb\}\right],$$

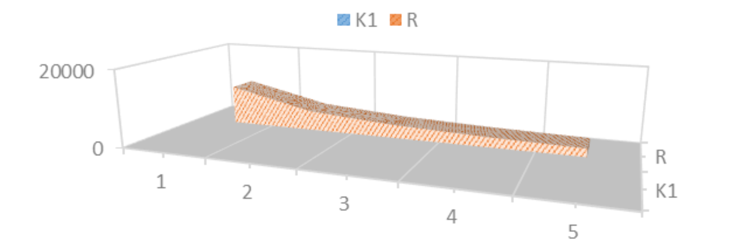
$$a_2 = (2A + b)\,b^2\left(\frac{1+K}{K_1} + Fb^2 + b\right) - \frac{KAb^3}{\in},$$

$$P_2 = \left[\frac{(2A+b)b^2}{\in} + b\left(\frac{1+K}{K_1} + Fb^2 + b\right)\{(2A + b)\,EP_r + mb\}\right] - \frac{KAb^2EP_r}{\in}.$$

From (8.1), we saying that σ_i^2 is positive, equation (8.1) for the sum of roots is positive, it is not possible if $f_1 > 0$ and $f_0 > 0$.
If $f_0 > 0$ and $f_1 > 0$ when $\bar{\delta} < \frac{\in}{A}$, $KEP_r < 2$, $K < 2 \in bF$.
Above conditions of the overstability.

9. Numerical Calculation

We show the graphical effect of different parameter in equation (6.9)

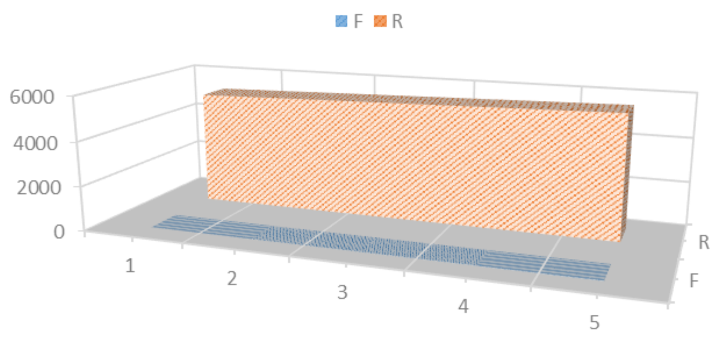


	1	2	3	4	5
K1	0.001	0.002	0.003	0.004	0.005
R	10342.18667	5222.186667	3515.52	2662.186667	2150.186667

FIGURE 1.

$$E = 1,\ \mathrm{P}_r = 2,\ \epsilon = 0.5,\ A = 0.1,\ F = 2,\ K = 0.2\ and\ \bar{\delta} = 0.05.$$

Figure 1 show that the variation of Rayleigh number R with respect to medium permeability i.e. medium permeability increases then Rayleigh number R decreases.

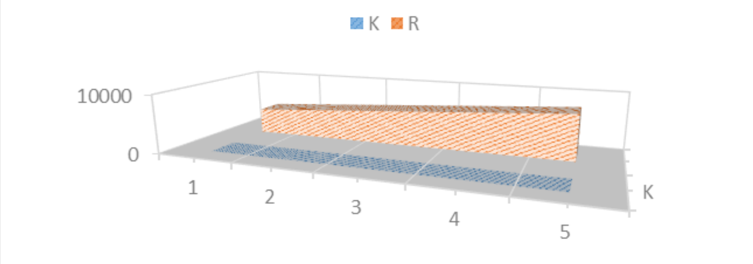


	1	2	3	4	5
F	0	3	6	9	12
R	5153.92	5256.32	5358.72	5461.12	5563.52

FIGURE 2.

$$E = 1,\ \mathrm{P}_r = 2,\ \epsilon = 0.5,\ A = 0.1,\ K = 0.2,\ K_1 = 0.002\ and\ \bar{\delta} = 0.05.$$

Figure 2 plot between Rayleigh number R and couple-stress parameter F i.e. couple-stress parameter F increases then Rayleigh number increases.

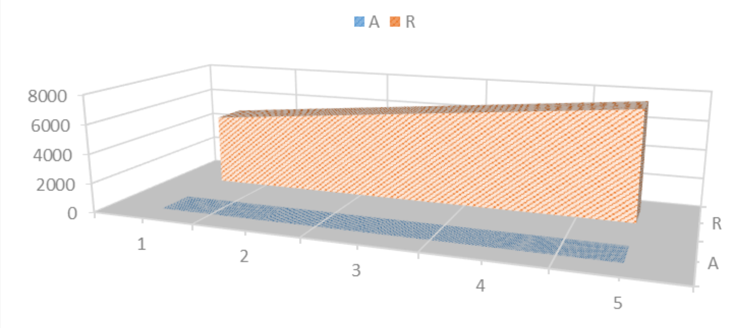


	1	2	3	4	5
K	0	0.2	0.4	0.6	0.8
R	4369.066667	5222.186667	6075.306667	6928.426667	7781.546667

FIGURE 3.

$$E = 1,\ P_r = 2,\ \in = 0.5,\ A = 0.1,\ F = 2,\ K_1 = 0.002\ and\ \bar{\delta} = 0.05.$$

Figure 3 shows the variation of Rayleigh number R with respect to coupling parameter K i.e. coupling parameter K increases then the Rayleigh number increases.



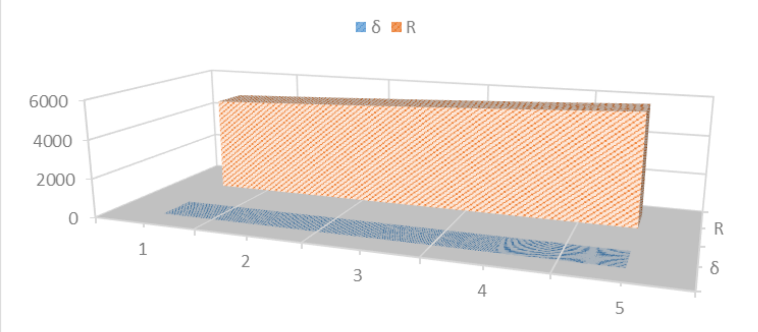
	1	2	3	4	5
A	0	0.2	0.4	0.6	0.8
R	4896	5548.373333	6200.746667	6853.12	7505.493333

FIGURE 4.

$$E = 1,\ P_r = 2,\ \in = 0.5,\ K = 0.2,\ F = 2,\ K_1 = 0.002\ and\ \bar{\delta} = 0.05.$$

Figure 4 represent the plot of Rayleigh number R versus micro polar coefficient A i.e. micro polar coefficient increases then the Rayleigh number increases.

Figure 5 plot between Rayleigh number R and heat conduction parameter $\bar{\delta}$ i.e. heat conduction parameter increases then Rayleigh number increases.



	1	2	3	4	5
δ	0	0.3	0.6	0.9	1.2
R	4895.8	5086.545455	5292.756757	5516.394366	5759.764706

FIGURE 5.

$$E = 1,\ \mathrm{P}_r = 2,\ \in = 0.5,\ K = 0.2,\ \ F = 2,\ K_1 = 0.002\, and\, A = 0.1.$$

10. Conclusions

According to the stationary convection and numerically discussion, we found that the effect of permeability is destabilizing. The effect of couple-stress parameter, coupling parameter, micro polar coefficient parameter and heat transfer parameter are stabilizing, among them the most important result that the effect of couple-stress parameter stabiliz the system.
The sufficient condition for the non-existence of over stability $\bar{\delta} < \frac{\in}{A}$, $KEP_r < 2$, $K < 2 \in bF$.

Author contributions:
Conceptualisation: Devilal Kumawat, Vijay Mehta, Vikas Tailor ; *Software:* Devilal Kumawat ; *Writing-Original Draft:* Devilal Kumawat

Conflicts of Interest: The authors declare that there are no conflicts of interest.

References

[1] S.A. Banyal and K. Singh, *A characterization of couple-stress fluid heated from below in a porous medium in the presence of a rotation*, International Journal of Advanced Computer and Mathematical Science, **3** (2012) 310-320.
[2] A.J. Chamkha, *Hydromagnetic two-phase flow in a channel*, International Journal of Engineering Science, **33** (1995) 437-446.
[3] A.J. Chamkha, T. Grosan and I. Pop, *Fully developed free convection of a micropolar fluid in a vertical channel*, International Communication in Heat and Mass Transfer, **29** (2002) 1119-1127.
[4] A.C. Eringen, *Simple microfluids*, International Journal of Engineering Science, **2** (1964) 205-217.

[5] A.C. Eringen, *Theory of micropolar fluid*, International Journal of Engineering Science, **16** (1966) 1-18.

[6] J.P. Kumar, J.C. Umavathi, A.J. Chamkha and I. Pop, *Fully-developed free-convective flow of micropolar and viscous fluids in a vertical channel*, Applied Mathematics Modelling, **34** (2010) 1175-1186.

[7] D. Kumawat and V. Mehta, *Numerical analysis of the micro-polar ferromagnetic rotating fluid flow saturating a porous medium*, Journal of Rajasthan Academy of Physical Science, **22** (2023) 57-71.

[8] D. Kumawat, R.D. Pankaj and V. Mehta, *Effect of the couple-stress on micro polar rotating fluid flow saturating a porous medium*, Journal of Computational Analysis and Applications, **31** (2023) 270-280.

[9] G. Lebon and C. Perez-Garcia, *Convective instability of a micropolar fluid layer by the method of energy*, International Journal of Engineering Science, **19** (1981) 1321-1329.

[10] R. Mittal and U.S. Rana, *Effect of dust particles on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium*, Applied Mathematics and Computation, **215** (2009) 2591-2607.

[11] S.K. Pundir, P.K. Nadiam and R. Pundir, *Effect of hall current on hydromagnetic instability of a couple-stress ferromagnetic fluid in the presence of varying gravitational field through a porous medium*, International Journal of Statistics and Applied Mathematics, **6** (2021) 01-15.

[12] C.R. Sharma and U. Gupta, *Thermal convection in micropolar fluids in porous medium*, International Journal of Engineering Science, **33** (1995) 1887-1892.

[13] B.K. Sharma, P.K. Sharma and S.K. Chauhan, *Effect of MHD on unsteady oscillatory couette flow through porous media*, International Journal of Applied Mechanics and Engineering, **24** (2022) 188-202.

[14] S. Shivakumara, S. Sureshkumar and N. Devaraju, *Effect of Non-uniform temperature gradients on the onset of convection in a couple-stress fluid-saturated porous medium*, Journal of Applied Fluid Mechanics, **5** (2012) 49-55.

[15] V.K. Stokes, *Couple-stress in fluid*, Physics of Fluid, **9** (1966) 1709-1715.

[16] Sunil, A. Sharma, P.K. Bharti and G.R. Shandil, *Effect of rotation on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium*, International Journal of Engineering Science, **44** (2006) 683-698.

[17] R.A. Wooding, *Rayleigh instability of a thermal boundary layer in flow through a porous medium*, Journal of Fluid Mechanics, **9** (1960) 183-192.

[18] P.Y. Xiong, M. Nazeer, F. Hussain, M.I. Khan, A. Saleem, S. Qayyum and Y.M. Chu, *Two-phase flow of couple stress fluid thermally effected slip boundary conditions: Numerical analysis with variable liquids properties*, Alexandria Engineering Journal, **61** (2022) 3821-3830.

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