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A STUDY OF THE COUPLE-STRESS ON MICRO POLAR ROTATING FLUID FLOW WITH THERMAL CONVECTION

By
Devilal Kumawat, Vijay Mehta and Madan Lal
Department of Mathematics and Statistics, Jai Narain Vyas University, Jodhpur, Rajasthan, India-342001
Email: devilalkumawat225@gmail.com, vijaymehtanrs@gmail.com and madan.lakhani288@gmail.com
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Abstract

The effect of couple-stress on micro-polar rotating fluid layer heated from below in the presence of uniform vertical magnetic field in a porous medium is studied. The dispersion relation has been analyzed, using normal mode and it is found that the medium permeability has destabilizing effect. The rotation, couple-stress parameter and micro-polar parameters have stabilizing effect. The sufficient condition for the non-existence of over stability has also been obtained.
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1 Introduction

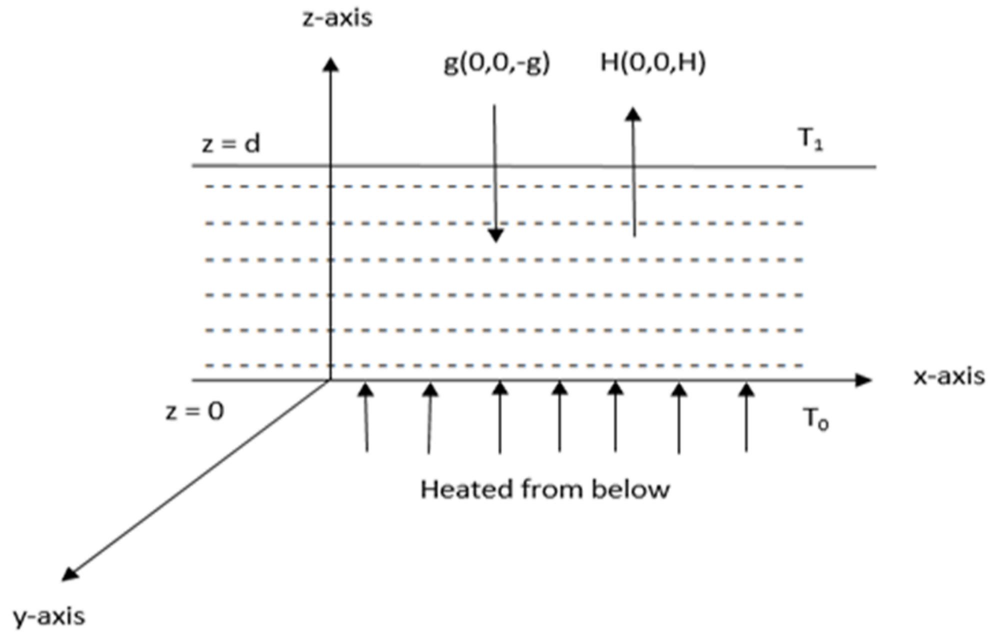
There are some important classes of fluid in technology areas, one of them being micro-polar fluid. In order to study micro-polar fluids, the general theory of micro polar fluid was introduced, micro rotational inertia and micro rotational effects of micro-polar fluid were shown according to Eringen [3, 4]. Qin and Kaloni [12] studied the problem of thermal instability in a rotating micro-polar fluid. Sharma and Gupta [14] investigated the thermal convection on micro polar fluid in porous medium. Sunil *et al.* [18] analyzed rotation and different parameters on a micro-polar ferromagnetic fluid flow. Kumawat *et al.* [6-9] analyzed the effect of the couple-stress, magnetic field, permeability, rotation and suspended particles on micro polar and visco-elastic fluid flow.

Stokes [17] studied the classical concept of couple-stress fluid. Kumar *et al.* [5] studied the rotation on thermal instability in couple-stress viscous elastic fluid. Banyal and Singh [2] investigated the rotation on the couple-stress fluid in a porous medium. Shivakumara *et al.* [16] used the Galerkin method to investigated the convection in a couple-stress fluid flow. Pundir *et al.* [10] analyzed the effect of permeability, couple-stress parameter and magnetization. Shah Zahir *et al.* [13] discussed the effect of couple stress on micro polar fluid flow with hall current. Sharma *et al.* [15] studied the effect of porosity, magnetic field and electrically conducting. Aparna *et al.* [1] investigated the couple stress fluid on rotating permeable sphere. Pei-Ying *et al.* [11] analyzed the couple stress fluid flow between parallel plates with thermal convection.

The application of this work in engineering science, chemical science and industry like as liquid crystal, colloids suspensions and clean engine lubricants. In this paper, we attempt to study of the couple-stress on micro-polar rotating fluid flow with thermal convection. In our knowledge this problem has not yet been investigated using the generalized Darcys model. In Section 2, we show the mathematical equations corresponding the model, in Section 3, we give the basic conditions, in section 4, we describe the linear perturbation equations, in Section 5 and Section 6, we find the boundary conditions and dispersion relation, in section 7, we present the effect of different parameters, in section 8, we find the conditions of over stability, in section 9, we numerically discuss the effect of different parameters.

2 Mathematical Formulation

An infinite, horizontal, incompressible micro-polar fluid layer of thickness d is assumed and has porosity $\in$ and medium permeability k_1. The upper limit $z = d$ and lower limit $z = 0$ are maintained at constant but varying temperatures T_0 and T_1 such that a study adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ has been continued. The rotation and gravity are applied along z-axis to the system.



The equations of continuity, momentum, internal angular momentum, temperature and basic's state are

$$\nabla.\vec{q} = 0, \tag{2.1}$$

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\in}\left(\vec{q}.\nabla\right)\vec{q}\right] = -\nabla P - \rho g \hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q} - \frac{1}{k_1}\left(\mu + \varsigma\right)\vec{q}$$

$$+\varsigma\left(\nabla \times \vec{v}\right) + \frac{2\rho_0}{\in}\left(\vec{q} \times \Omega\right) + \frac{\mu_e}{4\pi}\left(\nabla \times \vec{H}\right) \times \vec{H}, \tag{2.2}$$

$$\rho_0 J\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\in}\left(\vec{v}.\nabla\right)\vec{v}\right] = \left(\alpha' + \beta'\right)\nabla\left(\nabla.\vec{v}\right) + \gamma'\nabla^2\vec{v} + \frac{\varsigma}{\in}\left(\nabla \times \vec{q}\right) - 2\varsigma\vec{v}, \tag{2.3}$$

$$\left[\in \rho_0 C_v + \left(1 - \in\right)\rho_s C_s\right]\frac{\partial T}{\partial t} + \rho_0 C_v\left(\vec{q}.\nabla\right)T = \chi\,\nabla^2 T + \delta\left(\nabla \times \vec{v}\right).\nabla T, \tag{2.4}$$

$$\rho = \rho_0\left[1 - \alpha\left(T - T_a\right)\right]. \tag{2.5}$$

The Maxwell's equations are

$$\in \frac{\partial \vec{H}}{\partial t} = \nabla \times \left(\vec{q} \times \vec{H}\right) + \in \eta\nabla^2\vec{H}, \tag{2.6}$$

$$\nabla.\vec{H} = 0, \tag{2.7}$$

where, ρ - Fluid density, ρ_0 - Reference density, $\vec{q}$ - Filter velocity, $\vec{v}$ - Spin (micro rotation), μ - Shear kinematic viscosity coefficient, ς - Coupling viscosity coefficient, P - Pressure, μ' - Couple stress viscosity, $\hat{e}_z$ - Unit vector in z-direction, α' - Bulk spin viscosity coefficient, β' - Shear spin viscosity coefficient, γ' - Micro-polar viscosity coefficient, J - Micro inertia constant, t - time, C_v - Specific heat at constant volume, C_s - Specific heat of solid (Porous Material Matrix), ρ_s - Density of solid matrix, χ - Thermal conductivity, T - Temperature, δ - Micro-polar heat conduction coefficient, α - Coefficient of thermal expansion, $\vec{H} = (0, 0, H_z)$, H_z - Constant and η - Magnetic viscosity .

3 Basic State of Problem

The basic state is

$$\vec{q} = \vec{q}_b\left(0,0,0\right), \; \vec{v} = \vec{v}_b\left(0,0,0\right), \; \rho = \rho_b\left(z\right) \; and \, P = P_b\left(z\right).$$

From equations (2.1) to (2.7)

$$\frac{dP_b}{dz} + \rho_b g = 0, \tag{3.1}$$

$$T = T_b\left(z\right) = -\beta z + T_a, \tag{3.2}$$

$$\rho_b = \rho_0\left(1 + \alpha\beta z\right). \tag{3.3}$$

4 Linearize Perturbation Equations

$$\nabla.\vec{q}' = 0, \tag{4.1}$$

$$\frac{\rho_0}{\in}\frac{\partial \vec{q}'}{\partial t} = -\nabla P' - \rho' g\hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q}' - \frac{1}{k_1}\left(\mu + \varsigma\right)\vec{q}' + \varsigma\left(\nabla \times \vec{v}'\right) + \frac{2\rho_0}{\in}\left(\vec{q}' \times \Omega\right) + \frac{\mu_e}{4\pi}\left(\nabla \times \vec{h}\right) \times \vec{H}_b, \tag{4.2}$$

$$\rho_0 J\frac{\partial \vec{v}'}{\partial t} = \left(\alpha' + \beta'\right)\nabla\left(\nabla.\vec{v}'\right) + \gamma'\nabla^2\vec{v}' + \frac{\varsigma}{\in}\left(\nabla \times \vec{q}'\right) - 2\varsigma\vec{v}', \tag{4.3}$$

$$E\frac{\partial \theta}{\partial t} + \left(\vec{q}.\nabla\right)T_b = k_T\nabla^2\theta - \frac{\delta}{\rho_0 C_v}(\nabla \times \vec{v}')_z\beta + \beta(\vec{q}')_z, \tag{4.4}$$

$$\in \frac{\partial \vec{h}}{\partial t} = \nabla \times \left(\vec{q} \times \vec{h}\right) \times \vec{H}_b + \in \eta\nabla^2\vec{h} \tag{4.5}$$

$$\nabla.\vec{h} = 0, \tag{4.6}$$

$$\rho' = -\rho_0\alpha\theta. \tag{4.7}$$

Converting equations (4.1) to (4.6) by the following transforms:
$x = dx*$, $y = dy*$, $z = dz*$, $\vec{q}' = \frac{k_T}{d}\vec{q}*$, $P' = \frac{\mu k_T}{d^2}P*$, $\vec{v}' = \frac{k_T}{d^2}\vec{v}*$, $t = \frac{\rho_0 d^2}{\mu}t*$, $\nabla = \frac{\nabla*}{d}$, $\theta = \beta d\,\theta*$, we have

$$\nabla.\vec{q} = 0, \tag{4.8}$$

$$\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = -\nabla P + R\theta\hat{e}_z + \left(1 - F\nabla^2\right)\nabla^2\vec{q} - \frac{1}{K_1}\left(1 + K\right)\vec{q} + K\left(\nabla \times \vec{v}\right) + \frac{2}{\in}\left(\vec{q} \times \Omega\right) + Q\left(\nabla \times \vec{h}\right) \times \hat{e}_z, \tag{4.9}$$

$$\bar{J}\frac{\partial \vec{v}}{\partial t} = C_1\nabla\left(\nabla.\vec{v}\right) - C_0\nabla\left(\nabla \times \vec{v}\right) + K\left\{\frac{1}{\in}\left(\nabla \times \vec{q}\right) - 2\vec{v}\right\}, \tag{4.10}$$

$$EP_r\frac{\partial \theta}{\partial t} = \nabla^2\theta - \bar{\delta}(\nabla \times \vec{v})_z + (\vec{q})_z, \tag{4.11}$$

$$\in P_r\frac{\partial \vec{h}}{\partial t} = \frac{\partial \vec{q}}{\partial z} + \frac{\in P_r}{P_m}\nabla^2\vec{h}, \tag{4.12}$$

$$\nabla.\vec{h} = 0. \tag{4.13}$$

where, $R = \frac{\rho_0 g\alpha\beta d^4}{\mu k_T}$ - Thermal Rayleigh number, $P_r = \frac{\mu}{\rho_0 k_T}$ - Prandtl number, $F = \frac{\mu'}{\rho_0 d^2}$, $E = \in + \frac{(1-\in)\rho_s C_s}{\rho_0 C_v}$, $\bar{J} = \frac{J}{d^2}$, $K_1 = \frac{k_1}{d^2}$, $\bar{\delta} = \frac{\delta}{\rho_0 C_v d^2}$, $C_0 = \frac{\gamma'}{\mu d^2}$, $C_1 = \frac{\alpha' + \beta' + \gamma'}{\mu d^2}$, and $W = \vec{q}.\hat{e}_z$.

5 Boundary conditions

$$W = \frac{d^2W}{dz^2} = 0,\ \theta = 0\,at\,z = 0\,and\,z = d. \tag{5.1}$$

6 Dispersion Relation

Taking curl on both sides of equation (4.8), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t} + \left(\frac{1+K}{K_1}\right) - \left(1 - F\nabla^2\right)\nabla^2\right]\left(\nabla \times \vec{q}\right) = R\left(\frac{\partial \theta}{\partial y}\hat{e}_x + \frac{\partial \theta}{\partial x}\hat{e}_y\right) + K\nabla \times \left(\nabla \times \vec{v}\right) + \frac{2}{\in}\nabla \times \left(\vec{q} \times \Omega\right) + Q\frac{\partial}{\partial z}\left(\nabla \times \vec{h}\right). \tag{6.1}$$

Let $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $D = \frac{\partial}{\partial z}$, $\zeta_z = (\nabla \times \vec{q})_z$, ${\Omega_z}' = (\nabla \times \vec{v})_z$.
Taking curl and z-component of equations (6.1), (4.10) and (4.12), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t} + \left(\frac{1+K}{K_1}\right) - \left(1 - F\nabla^2\right)\nabla^2\right]\nabla^2 W = R\nabla_1^2\theta + K\nabla^2\,{\Omega_z}'\,\hat{e}_z - \frac{2}{\in}\Omega\left(D\zeta_z\right) + QD\left(\nabla^2\vec{h}_z\right), \tag{6.2}$$

$$\bar{J}\frac{\partial {\Omega_z}'}{\partial t} = C_0\nabla^2{\Omega_z}' - K\left[\frac{1}{\in}\nabla^2 W + 2{\Omega_z}'\right], \tag{6.3}$$

$$\in P_r\frac{\partial m_z}{\partial t} = D\zeta_z + \frac{\in P_r}{P_m}\nabla^2 m_z. \tag{6.4}$$

Taking z-component of equations (6.1), (4.11) and (4.12), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t}+\left(\frac{1+K}{K_1}\right)-\left(1-F\nabla^2\right)\nabla^2\right]\zeta_z=\frac{2}{\in}\Omega DW+QDm_z, \tag{6.5}$$

$$EP_r\frac{\partial\theta}{\partial t}=\nabla^2\theta-\bar{\delta}\,\Omega_z{}'+W, \tag{6.6}$$

$$\in P_r\frac{\partial\vec{h}_z}{\partial t}=DW+\frac{\in P_r}{P_m}\nabla^2\vec{h}_z. \tag{6.7}$$

Normal Mode Analysis

Let $\left[W,\,\zeta_z,\,\theta,\,\Omega_z{}',h_z,m_z\right]=\left[W\left(z\right),\,X\left(z\right),\,\Theta\left(z\right),\,G\left(z\right),B\left(z\right),M\left(z\right)\right]\exp.\left[i\,k_xx+i\,k_yy+\sigma t\right].$

Applying normal mode of equations (6.2) to (6.7), we have

$$\left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W$$
$$=-Ra^2\Theta+K\left(D^2-a^2\right)G-\frac{2}{\in}\Omega DX-QD\left(D^2-a^2\right)B, \tag{6.8}$$

$$\left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]X=\frac{2}{\in}\Omega DW+QDM, \tag{6.9}$$

$$\left[m\sigma+2A-\left(D^2-a^2\right)\right]G=-\frac{A}{\in}\left(D^2-a^2\right)W, \tag{6.10}$$

$$\left[EP_r\sigma-\left(D^2-a^2\right)\right]\Theta=-\bar{\delta}G+W, \tag{6.11}$$

$$\left[\in P_r\sigma-\frac{\in P_r}{P_m}\left(D^2-a^2\right)\right]M=DX, \tag{6.12}$$

$$\left[\in P_r\sigma-\frac{\in P_r}{P_m}\left(D^2-a^2\right)\right]B=DW, \tag{6.13}$$

where, $a^2=k_x^2+k_y^2$ - wave number, $\sigma=\sigma_r+i\,\sigma_r$ - stability parameter and $m=\frac{\bar{J}A}{K}$, $A=\frac{K}{C_0}$, A - ratio between the micro-polar viscous effect and micro-polar diffusion effects.
Now the boundary condition becomes

$$W=D^2W=0=X=DX=G,\Theta=0\,at\,z=0\,to\,z=1, \tag{6.14}$$

$$D^{2n}W=0\,at\,z=0\,to\,z=1,\ \text{where }n>0\ .$$

Thus, the proper solution satisfying (6.14) can be taken as

$$W=W_0\,\sin\pi z\,.$$

Eliminating Θ, G, Φ, X from (6.8) to (6.13) and putting the value of W *and* $b=\pi^2+a^2$, we have

$$b\left[\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)+Fb^2+b\right]\left[\left\{\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)+Fb^2+b\right\}\left\{P_r\sigma+\frac{P_r}{P_m}b\right\}\right.$$
$$\left.+\frac{Q\pi^2}{\in}\right]\left[m\sigma+2A+b\right]\left[EP_r\sigma+b\right]\left[P_r\sigma+\frac{P_r}{P_m}b\right]=Ra^2\left[\left\{\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)\right.\right.$$
$$\left.\left.+Fb^2+b\right\}\left\{P_r\sigma+\frac{P_r}{P_m}b\right\}+\frac{Q\pi^2}{\in}\right]\left[\left(m\sigma+2A+b\right)-\frac{\bar{\delta}Ab}{\in}\right]\left[P_r\sigma+\frac{P_r}{P_m}b\right]$$
$$+\frac{KAb^2}{\in}\left[\left\{\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)+Fb^2+b\right\}\left\{P_r\sigma+\frac{P_r}{P_m}b\right\}+\frac{Q\pi^2}{\in}\right]\left[EP_r\sigma+b\right]$$
$$\left[P_r\sigma+\frac{P_r}{P_m}b\right]-\frac{Q\pi^2b}{\in}\left[\left\{\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)+Fb^2+b\right\}\left\{P_r\sigma+\frac{P_r}{P_m}b\right\}+\frac{Q\pi^2}{\in}\right]$$
$$\left[m\sigma+2A+b\right]\left[EP_r\sigma+b\right]\ -\frac{4\Omega^2\pi^2}{\in^2}\left[P_r\sigma+\frac{P_r}{P_m}b\right]^2\left[m\sigma+2A+b\right]\left[EP_r\sigma+b\right]. \tag{6.15}$$

7 Stationary Convection

Putting $\rho = 0$ in equation (6.15), we have

$$R = \frac{1}{a^2\left[2A + b - \frac{\bar{\delta}Ab}{\in}\right]}\left[b^2(2A+b)\left(\frac{1+K}{K_1} + Fb^2 + b\right) - \frac{KAb^3}{\in}\right.$$
$$\left. + \frac{Q\pi^2 b}{\in}\frac{P_m}{P_r}(2A+b) + \frac{\frac{4\Omega^2\pi^2 b^2}{\in^2}\frac{P_r}{P_m}(2A+b)}{\left\{\left(\frac{1+K}{K_1} + Fb^2 + b\right)\frac{P_r}{P_m}b + \frac{Q\pi^2}{\in}\right\}}\right].$$

To investigate the behavior of medium permeability, rotation, couple-stress parameter, magnetic field, coupling parameter, micro-polar coefficient and micro-polar heat conduction parameter, we find the nature of $\frac{dR}{dK_1}$, $\frac{dR}{d\Omega}$, $\frac{dR}{dF}$, $\frac{dR}{dQ}$ $\frac{dR}{dK}$, $\frac{dR}{dA}$ and $\frac{dR}{d\bar{\delta}}$ respectively, then

$$\frac{dR}{dK_1} = \frac{-b^2(2A+b)(1+K)}{a^2{K_1}^2\left[2A + b - \frac{\bar{\delta}Ab}{\in}\right]}\left[1 - \frac{\frac{4\Omega^2\pi^2 b}{\in^2}\left(\frac{P_r}{P_m}\right)^2}{\left\{\left(\frac{1+K}{K_1} + Fb^2 + b\right)\frac{P_r}{P_m}b + \frac{Q\pi^2}{\in}\right\}^2}\right], \quad (7.1)$$

$$\frac{dR}{dK_1} < 0 \; if \, Q > \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m} \; and \, \bar{\delta} < \frac{\in}{A}.$$

From equation (7.1), we can say that the permeability has destabilizing effect when $Q > \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m} \; and \, \bar{\delta} < \frac{\in}{A}$.

$$\frac{dR}{d\Omega} = \frac{8\Omega\pi^2 b^2}{\in^2}\frac{P_r}{P_m}\frac{(2A+b)}{a^2\left\{\left(\frac{1+K}{K_1} + Fb^2 + b\right)\frac{P_r}{P_m}b + \frac{Q\pi^2}{\in}\right\}\left[2A + b - \frac{\bar{\delta}Ab}{\in}\right]}, \quad (7.2)$$

$$\frac{dR}{d\Omega} < 0 \, if \; \bar{\delta} < \frac{\in}{A}.$$

From equation (7.2) shows that the rotation has stabilizing effect when $\bar{\delta} < \frac{\in}{A}$.

$$\frac{dR}{dF} = \frac{b^4(2A+b)}{a^2\left[2A + b - \frac{\bar{\delta}Ab}{\in}\right]}\left[1 - \frac{\frac{4\Omega^2\pi^2}{\in^2}\left(\frac{P_r}{P_m}\right)^2}{\left\{\left(\frac{1+K}{K_1} + Fb^2 + b\right)\frac{P_r}{P_m}b + \frac{Q\pi^2}{\in}\right\}^2}\right], \quad (7.3)$$

$$\frac{dR}{dF} > 0 \; if \, Q > \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m} \; and \, \bar{\delta} < \frac{\in}{A}.$$

It is clear that the couple-stress parameter has stabilizing effect when $Q > \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m} \; and \, \bar{\delta} < \frac{\in}{A}$.

$$\frac{dR}{dQ} = \frac{\frac{\pi^2 b}{\in}\frac{P_m}{P_r}(2A+b)}{a^2\left[2A + b - \frac{\bar{\delta}Ab}{\in}\right]}\left[1 - \frac{\frac{4\Omega^2\pi^2}{\in^2}\left(\frac{P_r}{P_m}\right)^2}{\left\{\left(\frac{1+K}{K_1} + Fb^2 + b\right)\frac{P_r}{P_m}b + \frac{Q\pi^2}{\in}\right\}^2}\right], \quad (7.4)$$

$$\frac{dR}{dQ} > 0 \; if \, Q > \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m} \; and \, \bar{\delta} < \frac{\in}{A}.$$

It is clear that the magnetic field has stabilizing effect when $Q > \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m} \; and \, \bar{\delta} < \frac{\in}{A}$.

$$\frac{dR}{dK} = \frac{1}{a^2\left[2A + b - \frac{\bar{\delta}Ab}{\in}\right]}\left[Ab^2\left(\frac{2}{K_1} - \frac{b}{\in}\right)\frac{b^3}{K_1}\left\{1 - \frac{\frac{4\Omega^2\pi^2}{\in^2}\left(\frac{P_r}{P_m}\right)^2(2A+b)}{\left\{\left(\frac{1+K}{K_1} + Fb^2 + b\right)\frac{P_r}{P_m}b + \frac{Q\pi^2}{\in}\right\}^2}\right\}\right], \quad (7.5)$$

$$\frac{dR}{dK} > 0 \, if \, \frac{2}{K} > \frac{b}{\in}, \, Q > \frac{2\Omega\sqrt{(2A+b)}}{\pi}\frac{P_r}{P_m} \; and \, \bar{\delta} < \frac{\in}{A}.$$

Hence the coupling parameter has stabilizing effect when $\frac{2}{K} > \frac{b}{\in}$, $Q > \frac{2\Omega\sqrt{(2A+b)}}{\pi}\frac{P_r}{P_m} \; and \, \bar{\delta} < \frac{\in}{A}$.

$$\frac{dR}{dA} = \frac{1}{a^2\left[2A + b - \frac{\bar{\delta}Ab}{\in}\right]^2}\left[\frac{Kb^4}{\in}\left(\frac{\bar{\delta}}{K_1} - 1\right) + \frac{\bar{\delta}b^4}{\in}\left(\frac{1}{K_1} + Fb^2 + b\right)\right.$$
$$\left. + \frac{Q\pi^2\bar{\delta}b^3}{\in^2}\frac{P_m}{P_r} + \left\{\frac{\frac{4\Omega^2\pi^2 b^4\bar{\delta}}{\in^3}\frac{P_r}{P_m}}{\left\{\left(\frac{1+K}{K_1} + Fb^2 + b\right)\frac{P_r}{P_m}b + \frac{Q\pi^2}{\in}\right\}}\right\}\right], \quad (7.6)$$

$$\frac{dR}{dA} > 0 \; if \, \bar{\delta} > K_1.$$

From equation (7.6), we can say that the micro-polar coefficient has stabilizing effect when $\bar{\delta} > K\left(\frac{1+K}{K_1} + Fb^2 + b\right)$.

$$\frac{dR}{d\bar{\delta}} = \frac{\frac{Ab}{\in}}{a^2\left[2A+b-\frac{\bar{\delta}Ab}{\in}\right]^2}\left[Kb^3\left(\frac{1}{K_1}-\frac{A}{\in}\right)+\frac{2Ab^2K}{K_1}+b^2\left(2A+b\right)\right.$$
$$\left.\left(\frac{1}{K_1}+Fb^2+b\right)+\frac{Q\pi^2 b}{\in}\frac{P_m}{P_r}\left(2A+b\right)+\left\{\frac{\frac{4\Omega^2\pi^2b^2}{\in^2}\frac{P_r}{P_m}\left(2A+b\right)}{\left\{\left(\frac{1+K}{K_1}+Fb^2+b\right)\frac{P_r}{P_m}b+\frac{Q\pi^2}{\in}\right\}}\right\}\right], \quad (7.7)$$

$$\frac{dR}{d\delta} > 0 \; if \, \frac{1}{K_1} > \frac{A}{\in}.$$

From equation (7.7), shows that the micro-polar heat transfer parameter has stabilizing effect when $\frac{1}{K_1} > \frac{A}{\in}$.

8 Oscillatory Convection

Putting $\sigma = i\,\sigma_i$ in equation (6.15) then we get real and imaginary part, eliminating R between them, we have

$$f_0\sigma_i^8 + f_1\sigma_i^6 + f_2\sigma_i^4 + f_3\sigma_i^2 + f_4 = 0.$$

Put $s = \sigma_i^2$, we have

$$f_0 s^4 + f_1 s^3 + f_2 s^2 + f_3 s + f_4 = 0, \quad (8.1)$$

where,
$f_0 = a_1q_1 - p_1b_1$,
$f_1 = a_2q_1 + a_1q_2 - p_2b_1 - p_1b_2$,
$f_2 = a_3q_1 + a_2q_2 - p_3b_1 - p_2b_2 - p_1b_3$,
$f_3 = a_4q_1 + a_3q_2 - p_3b_2 - p_2b_3$,
$f_4 = a_4q_2 - p_3b_3$,

$$a_1 = -\frac{E{P_r}^2 mb}{\in^2},\; b_1 = \frac{m{P_r}^2a^2}{\in},$$

$$b_2 = -a^2\left[mP_r\left\{\frac{P_rb}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2}{\in}\right\}+\frac{{P_r}^2}{P_m}\frac{b}{\in}\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)\right.$$
$$\left.+\left\{P_r\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{P_r}{P_m}\frac{b}{\in}\right\}\left\{P_r\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)+\frac{P_r}{P_m}mb\right\}\right],$$

$$b_3 = a^2b\frac{P_r}{P_m}\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)\left\{b\frac{P_r}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2}{\in}\right\},$$

$$a_2 = E{P_r}^2m\left\{P_rb\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{2b^2}{\in}\frac{P_r}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)\right.$$
$$\left.+\frac{Q\pi^2b}{\in}\right\}+\left\{\frac{2P_rb}{\in}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b^2}{\in}\frac{P_r}{P_m}\right\}\left\{E{P_r}^2\left(2A+b\right)\right.$$
$$\left.+mb\left(P_r+\frac{E{P_r}^2}{P_m}\right)\right\}+\frac{P_rb}{\in}\left\{\left(2A+b\right)b\left(P_r+\frac{E{P_r}^2}{P_m}\right)+mb^2\frac{P_r}{P_m}\right\}$$
$$-\frac{KAb^2E{P_r}^3}{\in^2}+\frac{4\Omega^2\pi^2E{P_r}^3m}{\in^2}+\frac{Q\pi^2bE{P_r}^2m}{\in^2},$$

$$a_3 = -\left\{\frac{P_r}{P_m}b^2+\frac{Q\pi^2b}{\in}\right\}\left(\frac{1+K}{K_1}+Fb^2+b\right)\left\{E{P_r}^2\left(2A+b\right)\right.$$

$$+ mb\left(P_r+\frac{EP_r{}^2}{P_m}\right)\Bigg\} - \left\{P_r b\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{2b^2}{\in}\frac{P_r}{P_m}\right.$$

$$\left.\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2 b}{\in}\right\}\left\{(2A+b)\,b\left(P_r+\frac{EP_r{}^2}{P_m}\right)+mb^2\frac{P_r}{P_m}\right\}$$

$$-(2A+b)\,b^2\frac{P_r}{P_m}\left\{\frac{2P_r b}{\in}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b^2}{\in}\frac{P_r}{P_m}\right\} + \frac{KAb^2}{\in}\Big[EP_r$$

$$\left\{\frac{P_r b}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2}{\in}\right\}+\left\{P_r\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b^2}{\in}\frac{P_r}{P_m}\right\}$$

$$+\left(bP_r+\frac{EbP_r{}^2}{P_m}\right)+\frac{b^2}{\in}\frac{P_r{}^2}{P_m}\Bigg]-\frac{4\Omega^2\pi^2}{\in^2}\left[Emb^2\frac{P_r{}^3}{P_m{}^2}+2b\frac{P_r{}^2}{P_m}\left\{EP_r\,(2A+b)\right.\right.$$

$$\left.\left.+mb+(2A+b)\,bP_r{}^2\right]-\frac{Q\pi^2 b}{\in}\left[EmP_r\left\{\frac{P_r b}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2}{\in}\right\}\right.$$

$$\left.+\left\{P_r\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b}{\in}\frac{P_r}{P_m}\right\}\{EP_r\,(2A+b)+mb\}+\frac{P_r}{\in}\,(2A+b)\,b,\right.$$

$$a_4=(2A+b)\,b^2\frac{P_r}{P_m}\left(\frac{P_r b^2}{P_m}+\frac{Q\pi^2 b}{\in}\right)\left(\frac{1+K}{K_1}+Fb^2+b\right)-\frac{KAb^2}{\in}$$

$$\left[\frac{P_r b^2}{P_m}\left\{\frac{P_r b}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2}{\in}\right\}\right]+\frac{4\Omega^2\pi^2}{\in^2}\,(2A+b)\,b^3\left(\frac{P_r}{P_m}\right)^2$$

$$+\frac{Q\pi^2 b}{\in}\,(2A+b)\,b\left\{\frac{P_r b}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2}{\in}\right\},$$

$$P_1=\left[EP_r{}^2 m\left\{\frac{2P_r}{\in}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b^2}{\in}\frac{P_r}{P_m}\right\}\right.$$

$$\left.+\frac{P_r b}{\in}\left\{EP_r{}^2\,(2A+b)+mb\right\}\left(P_r+\frac{EP_r{}^2}{P_m}\right),\right.$$

$$P_2=-EP_r{}^2 m\left\{\frac{P_r}{P_m}b^2+\frac{Q\pi^2 b}{\in}\right\}\left(\frac{1+K}{K_1}+Fb^2+b\right)-\left\{P_r b\left(\frac{1+K}{K_1}\right.\right.$$

$$\left.\left.+Fb^2+b\right)+\frac{2b^2}{\in}\frac{P_r}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2 b}{\in}\right\}\left\{EP_r{}^2\,(2A+b)\right.$$

$$\left.+mb\left(P_r+\frac{EP_r{}^2}{P_m}\right)+\frac{b^3}{\in}\frac{P_r{}^2}{P_m}\,(2A+b)\right\}+\frac{KAb^2}{\in}\left[EP_r{}^2\left\{P_r\left(\frac{1+K}{K_1}\right.\right.\right.$$

$$\left.\left.\left.+Fb^2+b\right)+\frac{b}{\in}\frac{P_r}{P_m}\right\}+\frac{P_r}{\in}\left(bP_r+\frac{EbP_r{}^2}{P_m}\right)\right]-\frac{4\Omega^2\pi^2}{\in^2}\left[2Emb\frac{P_r{}^3}{P_m}\right.$$

$$\left.+P_r{}^2\left\{EP_r\,(2A+b)+mb\right\}\right]-\frac{Q\pi^2 b}{\in}\left[EmP_r\left\{P_r\left(\frac{1+K}{K_1}+Fb^2+b\right)\right.\right.$$

$$\left.\left.+\frac{b}{\in}\frac{P_r}{P_m}\right\}+\frac{P_r}{\in}\left\{EP_r\,(2A+b)+mb\right\},\right.$$

$$P_3=\left\{\frac{P_r}{P_m}b^2+\frac{Q\pi^2 b}{\in}\right\}\left(\frac{1+K}{K_1}+Fb^2+b\right)\left\{(2A+b)\,b\left(P_r+\frac{EP_r{}^2}{P_m}\right)+mb^2\frac{P_r}{P_m}\right\}$$

$$+\left\{P_r b\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{2b^2}{\in}\frac{P_r}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2 b}{\in}\right\}(2A+b)\,b^2\frac{P_r}{P_m}$$

$$-\frac{KAb^2}{\in}\left[\left(P_r b+\frac{EP_r{}^2 b}{P_m}\right)\left\{\frac{P_r b}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2}{\in}\right\}+\frac{P_r b^2}{P_m}\left\{P_r\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b}{\in}\frac{P_r}{P_m}\right\}\right]$$

$$+\frac{4\Omega^2\pi^2}{\in^2}\left[\frac{P_r{}^2 b^2}{P_m{}^2}\left\{EP_r\,(2A+b)+mb\right\}+2\,(2A+b)\,b\frac{P_r{}^2}{P_m}\right]$$

$$+\frac{Q\pi^2 b}{\in}\left[\left\{\frac{P_r b}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2}{\in}\right\}\{EP_r\,(2A+b)+mb\}+(2A+b)\,b\left\{P_r\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b}{\in}\frac{P_r}{P_m}\right\}\right],$$

$$q_1=-a^2\left[mP_r\left\{P_r\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{P_r}{P_m}\frac{b}{\in}\right\}+\frac{P_r}{\in}\left\{P_r\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)+\frac{P_r}{P_m}mb\right\}\right],$$

$$q_2=a^2\left[\left\{\frac{P_r b}{P_m}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{Q\pi^2}{\in}\right\}\left\{P_r\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)+\frac{P_r}{P_m}mb\right\}\right.$$
$$\left.+\frac{P_r b}{P_m}\left\{P_r\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b}{\in}\frac{P_r}{P_m}\right\}\left(2A+b-\frac{\bar{\delta}Ab}{\in}\right)\right].$$

From equation (8.1), we observed that $s=\sigma_i^2$ which is always positive, therefore the sum of roots equation of (8.1) is positive but this is impossible if $f_0>0$ and $f_1>0$, the sum of roots of equation (8.1) is $-{}^{f_1}/_{f_0}$
Thus, $f_0>0$ and $f_1>0$ are the sufficient condition for the non-existence of over stability.
Now $f_0>0$ and $f_1>0$ when $\bar{\delta}<\frac{\in}{A}$, $Q>\frac{Kb^2}{2\pi^2 E}\,and\,K=\frac{m\in^2}{A}$.

9 Numerical Calculation

Now, we show the numerically effect of different parameters employing equation (6.15)

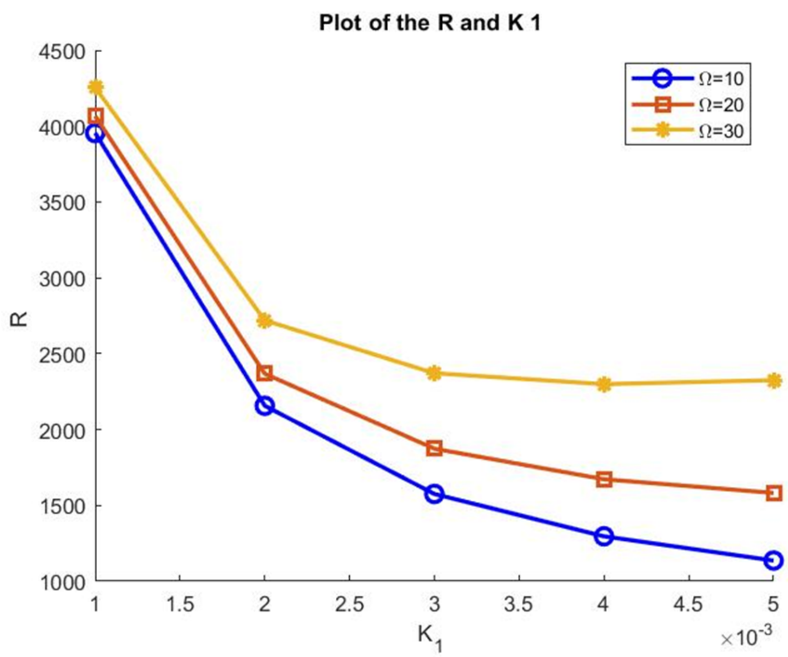


***Figure* 9.1**
$E=1,\ P_r=2,\ \in=0.5,\ A=0.1,\ F=2,\ Q=10, K=0.2,\ \Omega=10\,and\,\bar{\delta}=0.05.$

Fig 9.1 shows the variation of Rayleigh number R with respect to medium permeability K_1 i.e. medium permeability increases then the Rayleigh number decreases.

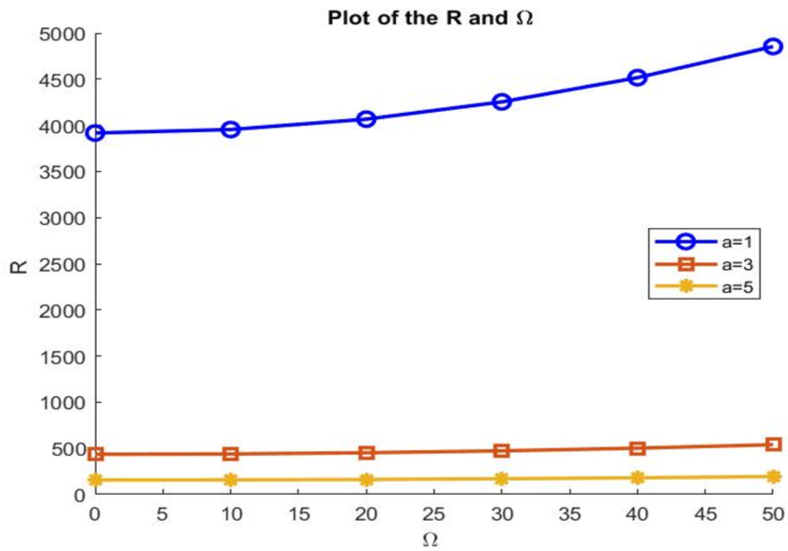


***Figure* 9.2**
$E = 1,\ P_r = 2,\ \in= 0.5,\ A = 0.1,\ F = 2,\ K = 0.2,\ K_1 = 0.002\, and\, \bar{\delta} = 0.05.$

Fig 9.2 represent the plot of Rayleigh number R versus rotation Ω i.e. rotation increases then the Rayleigh number increases.

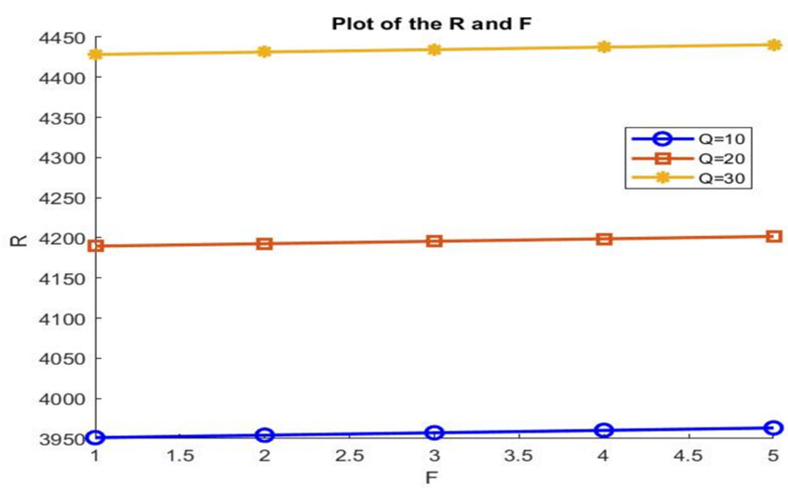


***Figure* 9.3**
$E = 1,\ P_r = 2,\ \in= 0.5,\ A = 0.1,\ \Omega = 10,\ K = 0.2,\ K_1 = 0.002\, and\, \bar{\delta} = 0.05.$

Fig 9.3 plot between Rayleigh number R and couple-stress parameter F i.e. couple-stress parameter increases then the Rayleigh number increases.

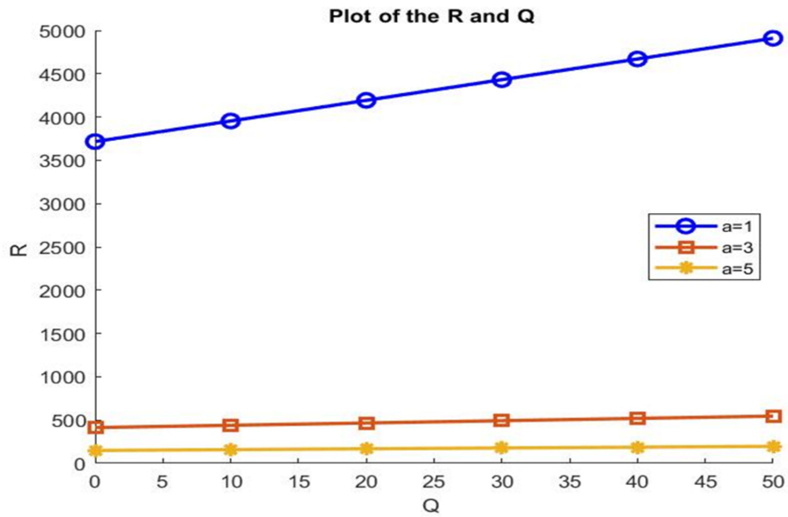


***Figure* 9.4**
$E = 1,\ P_r = 2,\ \in= 0.5,\ K = 0.2,\ \Omega = 10,\ F = 2,\ K_1 = 0.002\,and\,\bar{\delta} = 0.05.$

Fig 9.4 plot between Rayleigh number R and magnetic field Q i.e.magnetic field increases then the Rayleigh number R increases.

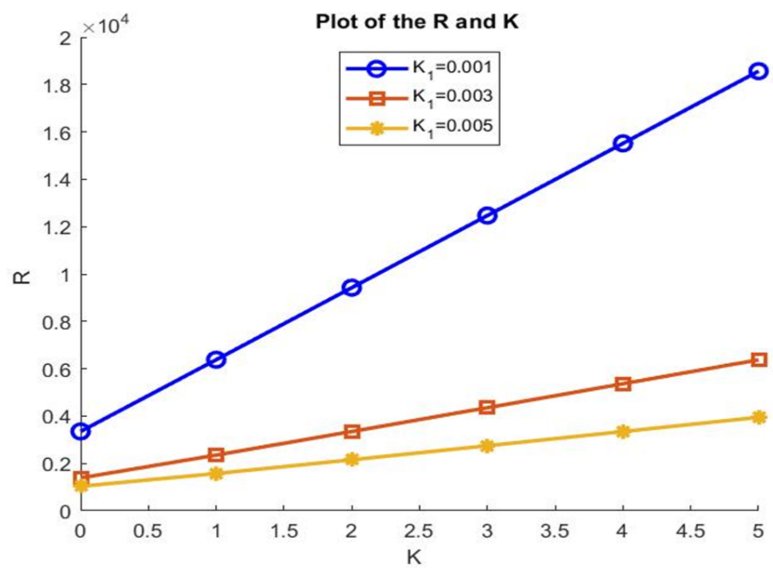


***Figure* 9.5**
$E = 1,\ P_r = 2,\ \in= 0.5,\ A = 0.1,\ \Omega = 10,\ F = 2,\ K_1 = 0.002\,and\,\bar{\delta} = 0.05.$

Fig 9.5 shows the variation of Rayleigh number R with respect to coupling parameter K i.e. coupling parameter increases then the Rayleigh number increases.

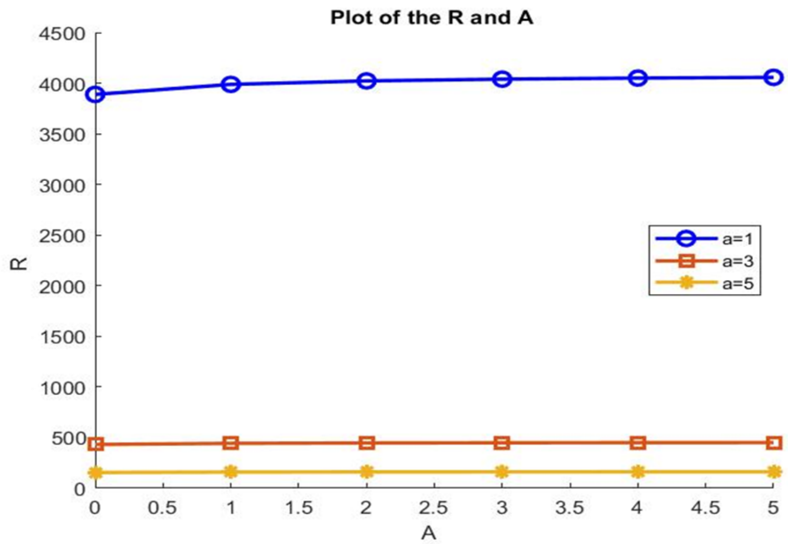


***Figure* 9.6**

$E = 1,\ P_r = 2,\ \in= 0.5,\ K = 0.2,\ \Omega = 10,\ F = 2,\ K_1 = 0.002\, and\, \bar{\delta} = 0.05.$

Fig 9.6 represent the plot of Rayleigh number R versus micro-polar coefficient A i.e. micro-polar coefficient increases then the Rayleigh number increases.

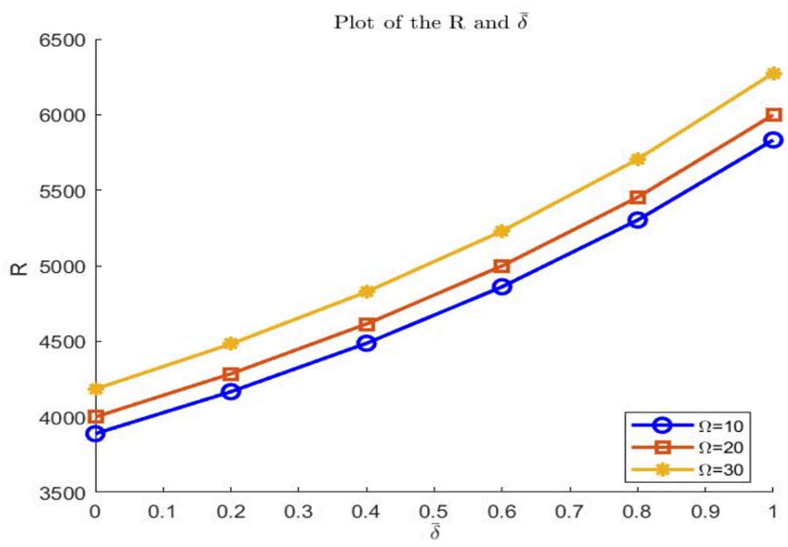


***Figure* 9.7**

$E = 1,\ P_r = 2,\ \in= 0.5,\ K = 0.2,\ \Omega = 10,\ F = 2,\ K_1 = 0.002\, and\, \bar{\delta} = 0.05.$

Fig 9.7 represent the plot of Rayleigh number R versus micro-polar heat conduction $\bar{\delta}$ i.e. micro-polar heat conduction increases then the Rayleigh number increases.

10 Conclusions

According to the stationary convection and numerically discussion we found that the effect of permeability is destabilizing. The effect of couple-stress parameter, rotation, magnetic field, coupling parameter, micro-polar coefficient and micro-polar heat conduction are stabilizing. Among them the most important result that the effect of rotation and magnetic field stabilizes on the system.

The sufficient condition for the non-existence of over stability is given by $\bar{\delta} < \frac{\in}{A}$, $Q > \frac{Kb^2}{2\pi^2 E}\, and\, K = \frac{m\in^2}{A}$.

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References

[1] P. Aparna and J.V.M. Ramana, Couple on a rotating permeable sphere in a couple stress fluid, *Ain Shams Engineering Journal*, **9** (2018),665-673.

[2] S.A. Banyal and K. Singh, A characterization of couple-stress fluid heated from below in a porous medium in the presence of a rotation, *International Journal of Advanced Computer and Mathematical Science*, **3**(3) (2012), 310-320.

[3] A.C. Eringen, Simple microfluids, *International Journal of Engineering Science*, **2**(2) (1964), 205-217.

[4] A.C. Eringen, Theory of micropolar fluid, *International Journal of Engineering Science*, **16**(1) (1966), 1-18.

[5] P. Kumar, R. Lal and P. Sharma, Effect of rotation on thermal instability in couple-stress elastio-viscous fluid, *Zeitschrift fur Naturforschung*, **59**(a) (2004), 407-411.

[6] D. Kumawat and V. Mehta, Numerical analysis of the micro polar ferromagnetic rotating fluid flow saturating a porous medium, *Journal of Rajasthan Academy of Physical Science*, **22**(1-2) (2023), 57-71.

[7] D. Kumawat, Instability of suspended particles on Visco-elastic rotating fluid flow with thermal convection, *European Bhemical Bulletin*, **12**(10) (2023), 4083-4094.

[8] D. Kumawat, V. Mehta and V. Tailor, Effect of the couple-stress on micro polar fluid flow saturating a porous medium, *Ganita*, **73**(2) (2023), 165-175.

[9] D. Kumawat, R.D. Pankaj and V. Mehta, Effect of the couple-stress on micro polar rotating fluid flow saturating a porous medium, *Journal of Computational Analysis and Applications*, **31**(2) (2023), 270-280.

[10] K.S. Pundir, K.P. Nadiam and R. Pundir, Effect of hall current on hydromagnetic instability of a couple-stress ferromagnetic fluid in the presence of varying gravitational field through a porous medium, *International Journal of Statistics and Applied Mathematics*, **6**(4) (2021), 01-15.

[11] X. Pie-Ying, N. Mubbashar, H. Farooq, M.I. Khan, A. Saleem, Q. Sumaira and C. Yu-Ming, Two-phase flow of couple stress fluid thermally effected slip boundary conditions: Numerical analysis with variable liquids properties, *Alexandria Engineering Journal*, **61** (2022), 3821-3830.

[12] Y. Qin and P.N. Kaloni, The thermal instability problem in a rotating micropolar fluid, *International Journal of Engineering Science*, **30**(9) (1992), 1117-1126.

[13] Z. Shah, P. Kumam, D. Abdullah, A.O. Ebraheem and T. Phatiphat, study of the couple stress convective micropolar fluid flow in a hall MHD generator system, *Frontiers in Physics*, **7** (2019), 171.

[14] C.R. Sharma and U. Gupta, Thermal convection in micropolar fluids in porous medium, *International Journal of Engineering Science*, **33**(13) (1995), 1887-1892.

[15] B.K. Sharma, P.K. Sharma and S.K. Chauhan, Effect of MHD on unsteady oscillatory couette flow through porous media, *International Journal of Applied Mechanics and Engineering*, **24**(1) (2022), 188-202.

[16] S. Shivakumara, S. Sureshkumar and N. Devaraju, Effect of Non-uniform temperature gradients on the onset of convection in a couple-stress fluid-saturated porous medium, *Journal of Applied Fluid Mechanics*, **5**(1) (2012), 49-55.

[17] V.K. Stokes, Couple-stress in fluid, *Physics of Fluid*, **9**(9) (1966), 1709-1715.

[18] A.S. Sharma, P.K. Bharti and G.R. Shandil, Effect of rotation on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium, *International Journal of Engineering Science*, **44** (2006), 683-698.