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Contents

Thermal Instability of the Couple-Stress on Micro Polar Fluid Flow

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Abstract. Effect of the couple-stress on micro polar fluid layer heated from below in a porous medium is studied. The dispersion relation was obtained using the normal mode and the problem has been numerically analyzed by MATLAB. The effect of permeability, couple-stress parameter, magnetic field and micro-polar parameters have been obtained and the effect of magnetic field on the system is very important result. The condition of over stability is also obtained.

Keywords: Micro-Polar Fluid · Couple-Stress · Thermal Convection · Porous Medium

2020 Mathematics Subject Classification: 76D05 · 76N10 35Q20 · 35Q30

1 Introduction

In order to study of micro-polar fluids, the general theory of micro polar fluid was introduced and discussed the micro rotational inertia and micro rotational effects of micro-polar fluid with investigated the problem of thermal convection in a rotating micro-polar fluid [1–3]. Some authors are discussed the Rayleigh instability [4–6] of flow via a porous medium and relaxing media. The effect of porosity, magnetic field and electrically conducting with different parameters on the micro-polar ferromagnetic fluid flow are studied [7–9].

The classical concept of couple-stress fluid and the rotation and couple-stress on viscoelastic fluid flow are investigated [10–12]. The analyzed the effect of the couple-stress on micro polar rotating fluid flow [13]. The Galerkin method [14,15] are used to investigate the convection in a couple-stress fluid flow. The effect of permeability, couple-stress parameter and magnetization are analyzed [16]. Tassaddiq et al. [17] discussed the a fractional model on generalized Casson fluid flow with Newtonian heating. Gupta et al. [18] analytical studied Williamson nanofluid flow with variable thickness, effects of nonlinear thermal radiation, and improved Fourier and Fick's laws.

This paper has some applications in geophysics, engineering science, chemical technology, astrophysics and industry like as liquid crystal, blood flows, colloids suspensions, contaminated and clean engine lubricants. This paper, we attempt

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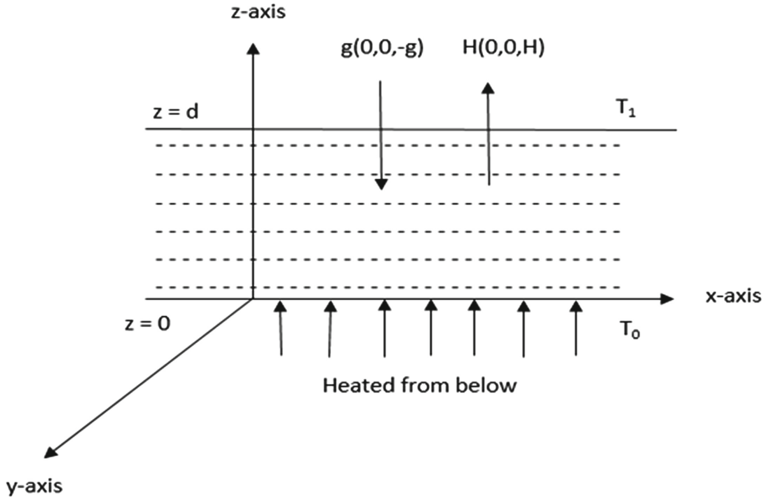
to study the thermal instability of couple-stress on micro-polar fluid flow. To our knowledge, Darcy's generalized model has not yet been used to study this problem.

2 Mathematical Formulation

A horizontal, infinite, incompressible electrically non-conducting fluid layer of thickness d is assumed, has porosity $\in$ and medium permeability k_1. The upper limit $z = d$ and lower limit $z = 0$ are maintained at constant but varying temperatures T_0 and T_1 such that a study adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ has been continued. The gravity is applied along with the z-axis of the system.



The equations of continuity, motion, angular momentum, temperature and basic's state are

$$\nabla.\vec{q} = o \tag{1}$$

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right] = -\nabla P - \rho g \hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q} - \frac{1}{k_1}(\mu + \varsigma)\vec{q} + \varsigma(\nabla \times \vec{v}) + \frac{\mu_e}{4\pi}\left(\nabla \times \vec{H}\right) \times \vec{H} \tag{2}$$

$$\rho_0 J\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\in}(\vec{v}.\nabla)\vec{v}\right] = (\alpha' + \beta')\nabla(\nabla.\vec{v}) + \gamma'\nabla^2\vec{v} + \frac{\varsigma}{\in}(\nabla \times \vec{q}) - 2\varsigma\vec{v} \tag{3}$$

$$[\in \rho_0 C_v + (1- \in)\rho_s C_s]\frac{\partial T}{\partial t} + \rho_0 C_v(\vec{q}.\nabla)T = \chi\nabla^2 T + \delta(\nabla \times \vec{v}).\nabla T \tag{4}$$

$$\rho = \rho_0 \left[1 - \alpha \left(T - T_a\right)\right] \tag{5}$$

The Maxwell's equation

$$\in \frac{\partial \vec{H}}{\partial t} = \nabla \times \left(\vec{q} \times \vec{H}\right) + \in \eta \nabla^2 \vec{H} \tag{6}$$

$$\nabla.\vec{H} = 0 \tag{7}$$

where, P - Pressure, ρ - Fluid density, ρ_0 - Reference density, μ - viscosity, $\vec{q}$ - velocity, ρ_s - Density of solid matrix, $\vec{v}$ - Spin, ς - Coupling viscosity coefficient, μ' - Couple stress viscosity,μ_e - Magnetic Permeability, $\hat{e}_z$ - Unit vector, α' - Bulk spin viscosity coefficient, β' - Shear spin viscosity coefficient, γ' - Micro-polar viscosity coefficient, C_v - Specific heat, C_s - Specific heat of solid, J - Micro inertia constant, t - time, χ - Thermal conductivity, T - Temperature, δ - Micro-polar heat conduction coefficient, α - Coefficient of thermal expansion, $\vec{H} = (0, 0, H_z)$,H_z - Constant η - Magnetic viscosity.

3 Basic State

$$\vec{q} = \vec{q}_b \left(0,0,0\right), \vec{v} = \vec{v}_b \left(0,0,0\right), \rho = \rho_b \left(z\right), P = P_b \left(z\right)$$

From Eqs. (1) to (7)

$$\frac{dP_b}{dz} + \rho_b g = 0 \tag{8}$$

$$T_b \left(z\right) = -\beta z + T_a \tag{9}$$

$$\rho_b = \rho_0 \left(1 + \alpha \beta z\right) \tag{10}$$

4 Linearize Perturbation Equations

$$\nabla.\vec{q}' = o \tag{11}$$

$$\frac{\rho_0}{\in}\frac{\partial \vec{q}'}{\partial t} = -\nabla P' - \rho' g \hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 \vec{q}' - \frac{1}{k_1}\left(\mu + \varsigma\right)\vec{q}' + \varsigma\left(\nabla \times \vec{v}'\right) + \frac{\mu_e}{4\pi}\left(\nabla \times \vec{h}\right) \times \vec{H}_b \tag{12}$$

$$\rho_0 J \frac{\partial \vec{v}'}{\partial t} = \left(\alpha' + \beta'\right)\nabla\left(\nabla.\vec{v}'\right) + \gamma' \nabla^2 \vec{v}' + \frac{\varsigma}{\in}\left(\nabla \times \vec{q}'\right) - 2\varsigma \vec{v}' \tag{13}$$

$$E\frac{\partial \theta}{\partial t} + \left(\vec{q}.\nabla\right) T_b = k_T \nabla^2 \theta - \frac{\delta}{\rho_0 C_v}\left(\nabla \times \vec{v}'\right)_z \beta + \beta(\vec{q}')_z \tag{14}$$

$$\in \frac{\partial \vec{h}}{\partial t} = \nabla \times \left(\vec{q}' \times \vec{H}_b\right) + \in \eta \nabla^2 \vec{h} \tag{15}$$

$$\nabla.\vec{h} = 0 \tag{16}$$

$$\rho' = -\rho_0 \alpha \theta \tag{17}$$

Converting Eq. (11) to (15) by the following transform $\vec{q}' = \frac{k_T}{d}\vec{q}*$, $P' = \frac{\mu k_T}{d^2}P*$, $\vec{v}' = \frac{k_T}{d^2}\vec{v}*$, $t = \frac{\rho_0 d^2}{\mu}t*$, $\nabla = \frac{\nabla *}{d}$, $\theta = \beta d\,\theta *$, we have

$$\nabla.\vec{q} = o \tag{18}$$

$$\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = -\nabla P + R\theta \hat{e}_z + \left(1 - F\nabla^2\right)\nabla^2 \vec{q} - \frac{1}{K_1}\left(1+K\right)\vec{q} + K\left(\nabla \times \vec{v}\right) + Q\left(\nabla \times \vec{h}\right) \times \hat{e}_z \tag{19}$$

$$\bar{J}\frac{\partial \vec{v}}{\partial t} = C_1 \nabla\left(\nabla.\vec{v}\right) - C_0 \nabla\left(\nabla \times \vec{v}\right) + K\left\{\frac{1}{\in}\left(\nabla \times \vec{q}\right) - 2\vec{v}\right\} \tag{20}$$

$$EP_r \frac{\partial \theta}{\partial t} = \nabla^2 \theta - \bar{\delta}(\nabla \times \vec{v})_z + (\vec{q})_z \tag{21}$$

$$\in P_r \frac{\partial \vec{h}}{\partial t} = \frac{\partial \vec{q}}{\partial z} + \frac{\in P_r}{P_m}\nabla^2 \vec{h} \tag{22}$$

where, $R = \frac{\rho_0 g \alpha \beta d^4}{\mu k_T}$ - Thermal Rayleigh number, $P_r = \frac{\mu}{\rho_0 k_T}$ - Prandtl number, $P_m = \frac{\mu}{\rho_0 \eta}$ - magnetic Prandtl number, $Q = \frac{{H_z}^2 d^2}{4\pi \mu k_T}$ - Chandrasekhar number, $F = \frac{\mu'}{\rho_0 d^2}$, $E = \in + \frac{(1-\in)\rho_s C_s}{\rho_0 C_v}$, $\bar{J} = \frac{J}{d^2}$, $K_1 = \frac{k_1}{d^2}$, $\bar{\delta} = \frac{\delta}{\rho_0 C_v d^2}$, $C_0 = \frac{\gamma'}{\mu d^2}$, $C_1 = \frac{\alpha' + \beta' + \gamma'}{\mu d^2}$ and $W = \vec{q}.\hat{e}_z$

5 Boundary Conditions

$$W = \frac{d^2 W}{dz^2} = 0,\ \theta = 0\, at\, z = 0\, and\, z = d \tag{23}$$

6 Dispersion Relation

Taking curl on both side Eq. (19), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t} + \left(\frac{1+K}{K_1}\right) - \left(1 - F\nabla^2\right)\nabla^2\right]\left(\nabla \times \vec{q}\right) = R\left(\frac{\partial \theta}{\partial y}\hat{e}_x + \frac{\partial \theta}{\partial x}\hat{e}_y\right) + K\nabla \times \left(\nabla \times \vec{v}\right) + Q\frac{\partial}{\partial z}\left(\nabla \times \vec{h}\right) \tag{24}$$

Assume $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $D = \frac{\partial}{\partial z}$, $\zeta_z = (\nabla \times \vec{q})_z$, $\Omega_z{}' = (\nabla \times \vec{v})_z$

Taking curl and z-component of Eqs. (24) and (20), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t} + \left(\frac{1+K}{K_1}\right) - \left(1 - F\nabla^2\right)\nabla^2\right]\nabla^2 W = R\nabla_1^2\theta + K\nabla^2\,\Omega_z{}'\,\hat{e}_z + QD\left(\nabla^2\vec{h}_z\right) \tag{25}$$

$$\bar{J}\frac{\partial \Omega_z{}'}{\partial t} = C_0\nabla^2\Omega_z{}' - K\left[\frac{1}{\in}\nabla^2 W + 2\Omega_z{}'\right] \tag{26}$$

Taking z-component of Eqs. (21) and (22), we have

$$EP_r\frac{\partial \theta}{\partial t} = \nabla^2\theta - \bar{\delta}\,\Omega_z{}' + W \tag{27}$$

$$\in P_r\frac{\partial \vec{h}_z}{\partial t} = DW + \frac{\in P_r}{P_m}\nabla^2\vec{h}_z \tag{28}$$

6.1 Normal Mode Analysis

Let $\left[W,\, h_z,\, \theta,\, \Omega_z{}'\right] = \left[W\left(z\right),\, B\left(z\right),\, \Theta\left(z\right),\, G\left(z\right)\right]\exp.\left[i\,k_x x + i\,k_y y + \sigma t\right]$
Apply normal mode of Eqs. (25) to (28), we have

$$\left[\frac{\sigma}{\in} + \left(\frac{1+K}{K_1}\right) + F\left(D^2 - a^2\right)^2 - \left(D^2 - a^2\right)\right]\left(D^2 - a^2\right)W = -Ra^2\Theta + K\left(D^2 - a^2\right)G - QD\left(D^2 - a^2\right)B \tag{29}$$

$$\left[m\sigma + 2A - \left(D^2 - a^2\right)\right]G = -\frac{A}{\in}\left(D^2 - a^2\right)W \tag{30}$$

$$\left[EP_r\sigma - \left(D^2 - a^2\right)\right]\Theta = -\bar{\delta}G + W \tag{31}$$

$$\left[\in P_r\sigma - \frac{\in P_r}{P_m}\left(D^2 - a^2\right)\right]B = DW \tag{32}$$

where, $a^2 = k_x^2 + k_y^2$ - wave number, $\sigma = \sigma_r + i\,\sigma_r$ - stability parameter and $m = \frac{\bar{J}A}{K}$, $A = \frac{K}{C_0}$, A - ratio between the micro-polar viscous effect and micro-polar diffusion effects.

$$W = D^2 W = 0 = X = DX = G, \Theta = 0\, at\, z = 0\ to\ z = 1 \tag{33}$$

$$D^{2n}W = 0\ at\ z = 0\ to\ z = 1, n > 0.$$

The solution of Eq. (33) is

$$W = W_0\,\sin \pi z$$

Eliminating Θ, G, B from (29) to (32) and put the value of $W\ and\ b = \pi^2 + a^2$, then we have

$$b\left[\frac{\sigma}{\in} + \left(\frac{1+K}{K_1}\right) + Fb^2 + b\right][m\sigma + 2A + b][EP_r\sigma + b]\left[P_r\sigma + \frac{P_r}{P_m}b\right] = Ra^2$$
$$[\ m\sigma + 2A + b) - \frac{\bar{\delta}Ab}{\in}\ ]\ [P_r\sigma + \frac{P_r}{P_m}b] + \frac{KAb^2}{\in}[EP_r\sigma + b]\left[P_r\sigma + \frac{P_r}{P_m}b\right]$$
$$- \frac{Q\pi^2 b^2}{\in}(m\sigma + 2A + b)[EP_r\sigma + b] \quad (34)$$

Now, the absence of magnetic field in Eq. (34), we get

$$b\left[\left\{\frac{\sigma}{\in} + \left(\frac{1+K}{K_1}\right) + Fb^2 + b\right\}\right][m\sigma + 2A + b][EP_r\sigma + b] = Ra^2$$
$$\left[(m\sigma + 2A + b) - \frac{\bar{\delta}Ab}{\in}\right] + \frac{KAb^2}{\in}[EP_r\sigma + b].$$

It is derived by Kumawat et al. [19].

Now, the absence of couple-stress in above equation, we have

$$b\left[\left\{\frac{\sigma}{\in} + \left(\frac{1+K}{K_1}\right)\right\}\right][m\sigma + 2A + b][EP_r\sigma + b] = Ra^2$$
$$\left[(m\sigma + 2A + b) - \frac{\bar{\delta}Ab}{\in}\right] + \frac{KAb^2}{\in}[EP_r\sigma + b].$$

This result derived by Sharma and Gupta [20].

7 Stationary Convection

Put the $\rho = 0$ in Eq. (34), then we have

$$R = \left[\frac{b^2(2A+b)\left(\frac{1+K}{K_1} + Fb^2 + b\right) - \frac{KAb^3}{\in} + \frac{Q\pi^2 b^2}{\in}\frac{P_m}{P_r}(2A+b)}{a^2\left[2A + b - \frac{\bar{\delta}Ab}{\in}\right]}\right] \quad (35)$$

To discuss the behavior of permeability, Couple-stress, Magnetic field, coupling parameter, micro-polar coefficient, micro-polar heat transfer parameter and find the nature of $\frac{dR}{dK_1}$, $\frac{dR}{dF}$, $\frac{dR}{dQ}$, $\frac{dR}{dK}$, $\frac{dR}{dA}$ and $\frac{dR}{d\bar{\delta}}$ respectively, we have

$$\frac{dR}{dK_1} = \frac{-b^2(2A+b)(1+K)}{a^2 {K_1}^2\left[2A + b - \frac{\bar{\delta}Ab}{\in}\right]} \quad (36)$$

$$\frac{dR}{dK_1} < 0\ if\ \bar{\delta} < \frac{\in}{A}$$

From Eq. (36), we can say that the permeability has destabilizing effect.

$$\frac{dR}{dF} = \frac{b^4\left(2A+b\right)}{a^2\left(\left(2A+b\right)-\frac{\bar{\delta}Ab}{\in}\right)} \tag{37}$$

$$\frac{dR}{dF} > 0 \; if \; \bar{\delta} < \frac{\in}{A}$$

It is clear that the couple-stress parameter has stabilizing effect.

$$\frac{dR}{dQ} = \frac{\frac{\pi^2 b^2 P_m}{P_r \in}\left(2A+b\right)}{a^2\left[2A+b-\frac{\bar{\delta}Ab}{\in}\right]} \tag{38}$$

$$\frac{dR}{dQ} < 0\, if \; \bar{\delta} < \frac{\in}{A}$$

Thus, the effect of magnetic field has stabilizing.

$$\frac{dR}{dK} = \frac{\left[Ab^2\left(\frac{2}{K_1}-\frac{b}{\in}\right)+\frac{b^3}{K_1}\right]}{a^2\left(\left(2A+b\right)-\frac{\bar{\delta}Ab}{\in}\right)} \tag{39}$$

$$\frac{dR}{dK} > 0 \; if \; \frac{2}{K} > \frac{b}{\in} \; and \; \bar{\delta} < \frac{\in}{A}$$

Hence, we can say that the coupling parameter has stabilizing effect.

$$\frac{dR}{dA} = \frac{\frac{Kb^4}{\in}\left\{\frac{\bar{\delta}}{K_1}-1\right\}+\frac{\bar{\delta}b^4}{\in}\left(\frac{1}{K_1}+Fb^2+b\right)+\frac{Q\pi^2\bar{\delta}b^4}{\in}\frac{P_m}{P_r}}{a^2\left[\left(2A+b\right)-\frac{\bar{\delta}Ab}{\in}\right]^2} \tag{40}$$

$$\frac{dR}{dA} > 0 \; if \, \bar{\delta} > K_1$$

It is clear that the effect of micro-polar coefficient has stabilizing.

$$\frac{dR_1}{d\bar{\delta}} = \frac{Ab}{\in}\left[\frac{Ab^3\left(2-\frac{K}{\in}\right)+b^4+b^2\left(2A+b\right)\left(\frac{1+K}{K_1}+Fb^2\right)+\frac{Q\pi^2b^2}{\in}\frac{P_m}{P_r}\left(2A+b\right)}{a^2\left[2A+b-\frac{\bar{\delta}Ab}{\in}\right]^2}\right] \tag{41}$$

$$\frac{dR}{d\bar{\delta}} > 0\, if\, K < 2\in$$

From Eq. (41), we can say that the effect of micro-polar heat transfer parameter has stabilizing.

8 Oscillatory Convection

Putting $\sigma = i\,\sigma_i$ in Eq. (34), then we get real and imaginary part, eliminating R between them, we have

$$f_0\sigma_i^4 + f_1\sigma_i^2 + f_2 = 0$$

Putting $s = \sigma_i^2$, we have

$$f_0 s^2 + f_1 s + f_2 = 0 \tag{42}$$

where,

$$f_0 = a_1 q_1 - p_1 b_1,$$
$$f_1 = a_2 q_1 - p_2 b_1 - p_1 b_2,$$
$$f_2 = a_3 q_1 - p_2 b_2,$$

$$a_1 = \frac{E{P_r}^2 mb}{\in}, \quad b_1 = -m\mathrm{P}_r a^2 \quad and \quad b_2 = a^2 b\frac{P_r}{P_m}\left(2A + b - \frac{\bar{\delta}Ab}{\in}\right)$$

$$a_2 = -\left[E{P_r}^2 b\,(2A+b)\left(\frac{1+K}{K_1} + Fb^2 + b\right) + \left(bP_r + \frac{Eb{P_r}^2}{P_m}\right)\right.$$
$$\left.\left\{mb\left(\frac{1+K}{K_1} + Fb^2 + b\right) + \frac{b}{\in}(2A+b)\right\} + \frac{mb^3}{\in}\frac{P_r}{P_m} - \frac{KAb^3E{P_r}^2}{\in} + \frac{Q\pi^2 b^2 EP_r m}{\in}\right],$$

$$a_3 = \frac{P_r}{P_m}(2A+b)\,b^2\left(\frac{1+K}{K_1} + Fb^2 + b\right) - \frac{KAb^3}{\in}\frac{P_r}{P_m}b^2 + \frac{Q\pi^2 b^3(2A+b)}{\in},$$

$$P_1 = -\left[E{P_r}^2\left\{mb\left(\frac{1+K}{K_1} + Fb^2 + b\right) + \frac{b}{\in}(2A+b)\right\} + \frac{mb}{\in}\left(bP_r + \frac{Eb{P_r}^2}{P_m}\right)\right],$$

$$P_2 = \left[(2A+b)\,b\left(\frac{1+K}{K_1} + Fb^2 + b\right)\left(bP_r + \frac{Eb{P_r}^2}{P_m}\right) + \frac{P_r}{P_m}b^2\right.$$
$$\left.\left\{mb\left(\frac{1+K}{K_1} + Fb^2 + b\right) + \frac{b}{\in}(2A+b)\right\}\right] - \frac{KAb^3}{\in}\left(bP_r + \frac{Eb{P_r}^2}{P_m}\right)$$
$$+ \frac{Q\pi^2 b^2}{\in}\left\{(2A+b)\,EP_r + mb\right\},$$

$$q_1 = a^2\left[P_r\left(2A + b - \frac{\bar{\delta}Ab}{\in}\right) + mb\frac{P_r}{P_m}\right].$$

From (42), we saying that $s = \sigma_i^2$ is positive, Eq. (42) for the sum of roots is positive, it is not possible if $f_1 > 0$ and $f_0 > 0$.
If $f_1 > 0$ and $f_0 > 0$ when $\bar{\delta} < \frac{\in}{A}$, $K > 2m\in$, $Q < \frac{1}{\pi^2 bP_m}$ $and\, K < 2$.
Above conditions of the overstability.

9 Numerical Calculation by MATLAB

Now, we discuss the effect of different parameters from Eq. (34) by MATLAB.

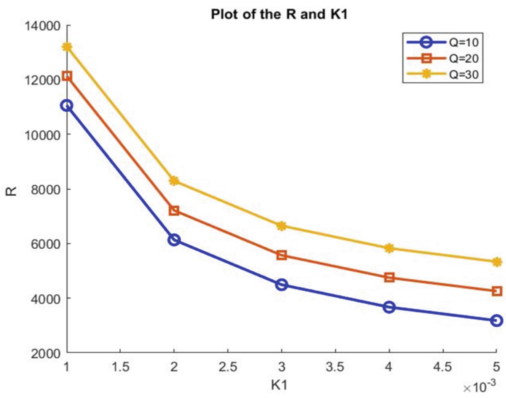


Fig. 1. Plot of Rayleigh number R and medium permeability K_1.
$E = 1$, $\mathrm{P}_r = 2$, $\in = 0.5$, $A = 0.5$, $K = 0.2$, $F = 2\, Q = 10$ and $\bar{\delta} = 0.05$.

Figure 1 shows that the medium permeability K_1 increases then the Rayleigh number R decreases.

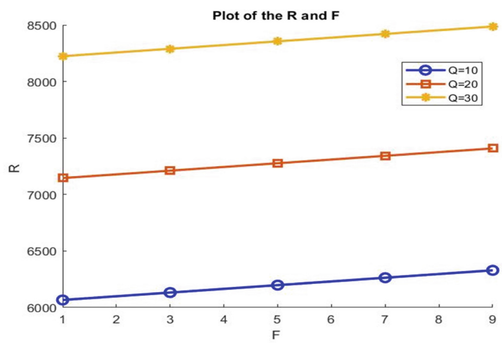


Fig. 2. Plot between Rayleigh number and couple-stress.
$E = 1$, $\mathrm{P}_r = 2$, $\in = 0.5$, $A = 0.5$, $K = 0.2$, $K_1 = 0.002$ and $\bar{\delta} = 0.05$.

Figure 2 represent that the couple-stress F increases then the Rayleigh number R increases.

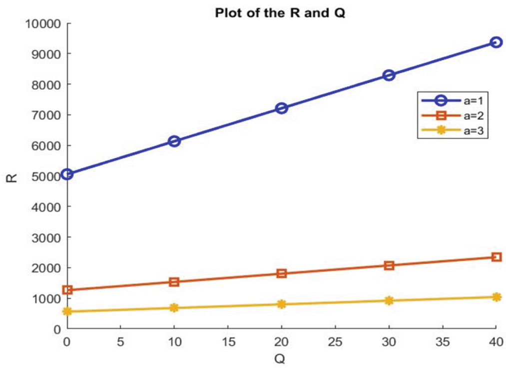


Fig. 3. Plot of the Rayleigh number and magnetic field.
$E = 1,\ \mathrm{P}_r = 2,\ \in= 0.5,\ K = 0.2,\ \ F = 2,\ K_1 = 0.002$ and $\bar{\delta} = 0.05$.

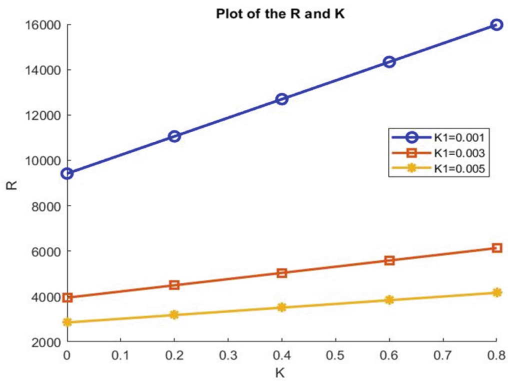


Fig. 4. Plot of Rayleigh number R and coupling parameter K.
$E = 1,\ \mathrm{P}_r = 2,\ \in= 0.5,\ K = 0.2,\ \ F = 2,\ K_1 = 0.002$ and $\bar{\delta} = 0.05$.

Figure 3 denote that the magnetic field increases then the Rayleigh number R increases.

Figure 4 represent that the coupling parameter K increases then the Rayleigh number R increases.

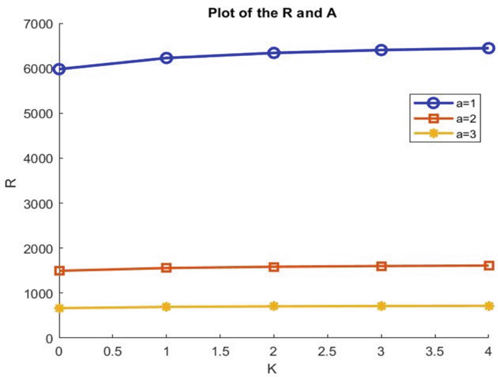


Fig. 5. Plot between Rayleigh number R and micro-polar coefficient A. $E = 1,\ \mathrm{P}_r = 2,\ \in= 0.5,\ K = 0.2,\ \ F = 2,\ K_1 = 0.002$ and $Q = 10$.

Figure 5 represent that the micro-polar coefficient A increases then the Rayleigh number R increases.

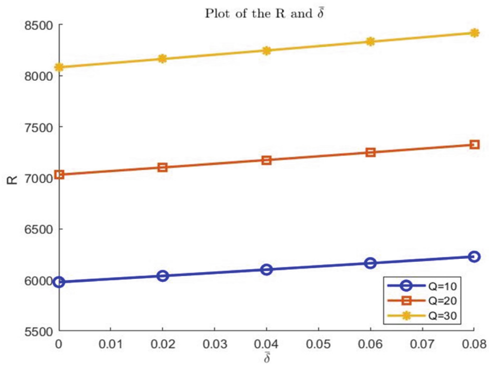


Fig. 6. Plot of the Rayleigh number R and heat conduction parameter. $E = 1,\ \mathrm{P}_r = 2,\ \in= 0.5,\ K = 0.2,\ \ F = 2,\ K_1 = 0.002$ and $A = 0.5$.

Figure 6 denote that the heat conduction parameter increases then the Rayleigh number R increases.

10 Conclusions

According to numerically discussion and stationary convection, we found that the effect of medium permeability is destabilizing. The effect of couple-stress, magnetic field, coupling parameter, micro polar coefficient and micro polar heat transfer parameter are stabilizing. Among them the most important result that the effect of magnetic field stabilize on the system.

$$\bar{\delta} < \frac{\in}{A}, K > 2m \in, Q < \frac{1}{\pi^2 b P_m} \, and \, K < 2.$$

Above conditions of the overstability.

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